Solving Equations

Properties of Equality

Addition Property of Equality: you can add the same number to both sides of the equation, and the statement remains true.

$$3 = 3$$
$$3+2 = 3+2$$
$$5 = 5$$

$$a = b$$
$$a+c = b+c$$

Subtraction Property of Equality- you can subtract the same number from both sides of the equation...

$$5 = 5$$
$$5-1 = 5-1$$
$$4 = 4$$

$$a = b$$
$$a-c = b-c$$

Multiplication Property of Equality- you can multiply both sides of the equation by the same number...

$$a = b$$
$$c(a) = c(b)$$

$$7 = 7$$
$$2(7) = 2(7)$$
$$14 = 14$$

Division Property of Equality- you can divide both sides of an equation by the same nonzero number...

$$\frac{a}{c} = \frac{b}{c} \quad c \neq 0$$

$$\frac{8}{2} = \frac{8}{2}$$

$4 = 4$

***Distributive Property $a(b+c) = ab + ac$

Ex.1 Solve the equation using Properties of Equality.

a) $3x - \cancel{2} = 6$ APE

$\quad\ \cancel{+2} \quad +2$

$\frac{\cancel{3}x}{\cancel{3}} = \frac{8}{3}$ DPE

$\boxed{x = \frac{8}{3}}$

b) $\frac{1}{2}z + \cancel{4} = 10$ SPE

$\quad\ \cancel{-4} \quad -4$

$\cancel{2}\left(\cancel{\tfrac{1}{2}}z\right) = (6)2$ MPE

$\boxed{z = 12}$

Ex.2 Write an equation and solve for the unknown quantity.

c) An ostrich that is 108 inches tall is 20 inches taller than 4 times the height of a kiwi in inches.

$108 \text{ in.} = 4x + \cancel{20}$

$-20 \qquad\qquad \cancel{-20}$

$$\frac{\quad}{4} = \frac{\quad}{4}$$

$$\boxed{x=22}$$

d) An emu that measures 60 inches in height is 70 inches less than 5 times the height of a kakapo. What is the height of a kakapo?

$$60 = 5x - \cancel{70}$$
$$+70 \qquad \cancel{+70}$$

$$\frac{130}{5} = \frac{\cancel{5}x}{\cancel{5}}$$

$$\boxed{26 = x}$$

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Writing equations

Ex.1 One half an amount added to $50 is $262.

$$\frac{1}{2}x + \cancel{50} = 262$$
$$-\cancel{50} \quad -50$$
$$\frac{2}{1}\cdot\frac{1}{2}x = 212 \cdot 2$$
$$\boxed{x = 424}$$

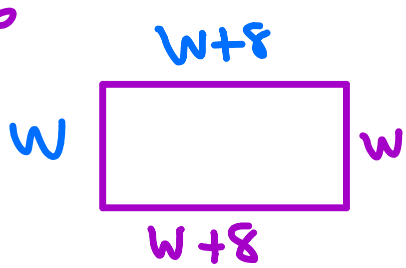
Ex.2 Four times the sum of a number and 10 is 48.

$$4(x+10) = 48$$
$$4x + \cancel{40} = 48$$
$$-\cancel{40} \quad -40$$
$$\frac{4x}{4} = \frac{8}{4}$$
$$\boxed{x = 2}$$

$$\frac{4(x+10)}{4} = \frac{48}{4}$$
$$x + \cancel{10} = 12$$
$$-\cancel{10} \quad -10$$
$$\boxed{x = 2}$$

Ex.3 A rectangular garden is fenced on all sides with 256 feet of fencing. The garden is 8 feet longer than it is wide. Find the length and width of the garden.

$P = 256$



$$w+8+w+w+8+w = 256$$
$$4w + \cancel{16} = 256$$
$$-\cancel{16} \quad -16$$
$$\frac{4w}{4} = \frac{240}{4}$$
$$\boxed{w = 60}$$

Ex.4 One moving company charges \$800 plus \$16 per hour. Another company charges \$720 plus \$21 per hour. At what number of hours will the charge by both companies be the same?

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Solving for a variable

- Literal equations- are equations that contain two or more variables. Many literal equations are formulas.

Ex.1 solve for the indicated variable.

a) $\frac{V}{lw} = \frac{\cancel{l} \cdot w \cdot h}{\cancel{lw}}$

$\boxed{h = \frac{V}{lw}}$

b) $V(D) = \left(\frac{m}{\cancel{V}}\right)\cancel{V}$

$$\boxed{m = VD}$$

c) $2(A) = \left(\cancel{\tfrac{1}{2}}(a+b)h\right)\cancel{2}$

$$\frac{2A}{a+b} = \frac{\cancel{(a+b)}h}{\cancel{(a+b)}}$$

$$\boxed{h = \frac{2A}{a+b}}$$

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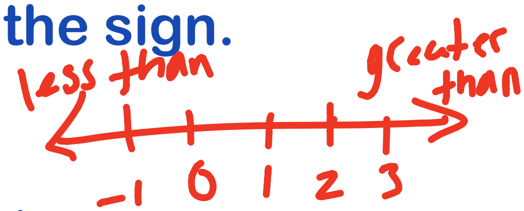
pg. 58 #1-20 odd

Creating and Solving Inequalities

- when you multiply or divide by a negative, it changes the direction of the sign.



- Is less than $<$
- Is less than or equal to $\leq$
- Is greater than $>$
- Is greater than or equal to $\geq$
- Is not equal to $\neq$

Ex.1 solve the inequality

a) $2x \leq -2(4x+4)$

$2x \leq -8x - 8$

$+8x \quad +8x$

$\frac{10x}{10} \leq \frac{-8}{10}$

$x \leq -\frac{8}{10}$

$\boxed{x \leq -\frac{4}{5}}$

$\frac{2x}{-2} \leq \frac{-2(4x+4)}{-2}$

$-1x \geq 4x + 4$

$-4x \quad -4x$

$\frac{-5x}{-5} \geq \frac{4}{-5}$

$\boxed{x \leq -\frac{4}{5}}$

b) $\frac{1}{2}(-2x-12) > 4-6x$

$2 \cdot \frac{1}{2}(-2x-12) > (4-6x)2$

$$-1x - 6 > 4 - \cancel{6x}$$
$$+6x \qquad +\cancel{6x}$$
$$5x - \cancel{6} > 4$$
$$+\cancel{6} \qquad +6$$
$$\frac{\cancel{5}x}{\cancel{5}} > \frac{10}{5}$$
$$\boxed{x > 2}$$

$$-2x - 12 > 8 - \cancel{12x}$$
$$+12x \qquad +\cancel{12x}$$
$$10x - \cancel{12} > 8$$
$$+\cancel{12} \qquad +12$$
$$\frac{\cancel{10}x}{\cancel{10}} > \frac{20}{10}$$
$$\boxed{x > 2}$$

Ex.2 writing inequalities.

a) It costs $20 to attend a play. A seasons pass cost $180. For what number of plays is it cheaper to pay $20 than to buy a seasons pass?

$$\frac{\cancel{20}x}{\cancel{20}} < \frac{180}{20}$$
$$x < 9$$

b) the sum of two consecutive integers is less than 83. Find the pair of integers with the greatest sum.

$$x + (x+1) < 83$$

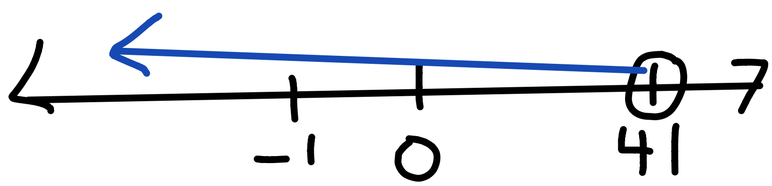
$$2x + \cancel{1} < 83$$

$$\quad -\cancel{1} \quad\quad -1$$

$$\frac{\cancel{2}x}{\cancel{2}} < \frac{82}{2}$$

$$x < 41$$

$$\boxed{40, 41}$$



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Creating and Solving Compound Statements

- Graphs of compound inequalities involving AND is the intersection of simple inequality graphs. $x > 2$ $\quad x < 6$
- Graphs of compound inequalities involving OR is the union of simple inequality graphs.

$x < 2$ $\quad x > 6$

-

Ex.1 Solve each compound inequality

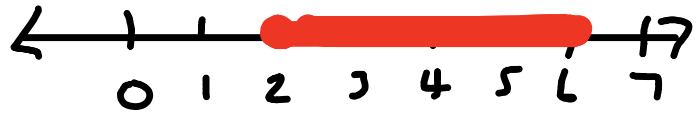
a) $4 \le x + \cancel{2} \le 8$

$-2 \qquad -\cancel{2} \qquad -2$

$2 \le x \le 6$

$x \le 6$

$x \ge 2$



b) $-5 \le 2x + \cancel{3} < 9$

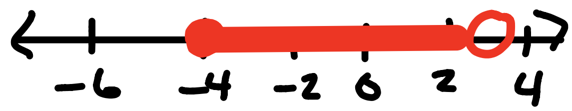
$-3 \qquad -\cancel{3} \qquad -3$

$\frac{-8}{2} \le \frac{\cancel{2}x}{\cancel{2}} < \frac{6}{2}$

$-4 \le x < 3$

$x < 3$

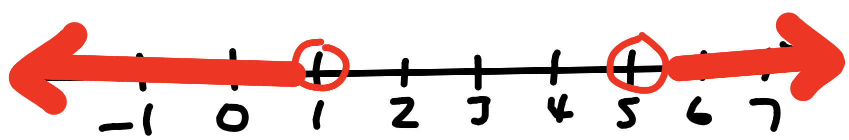
$x \ge -4$



c) $-\cancel{4} + x > 1$ OR $-\cancel{4} + x < -3$

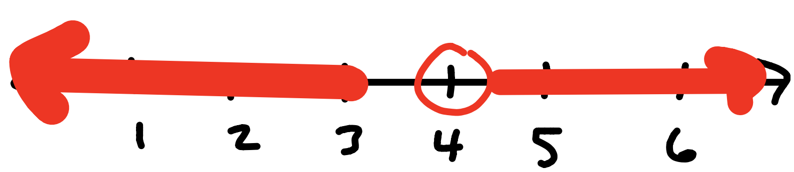
$+\cancel{4} \qquad +4 \qquad\qquad +\cancel{4} \qquad +4$

$\boxed{x > 5} \qquad\qquad \boxed{x < 1}$



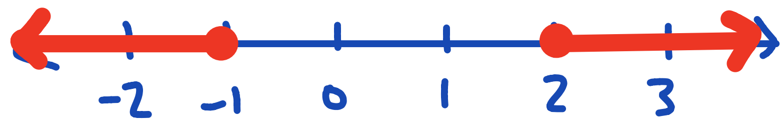
d) $\frac{\cancel{2}x}{\cancel{2}} \le \frac{6}{2}$ OR $\frac{\cancel{3}x}{\cancel{3}} > \frac{12}{3}$

$\boxed{x \le 3} \qquad\qquad \boxed{x > 4}$



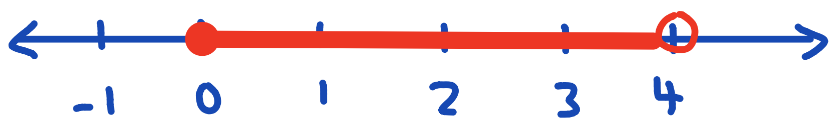
Ex.2 write the compound inequality

a)



$x \leq -1$ or $x \geq 2$

b)



$x \geq 0$ and $x < 4$

Pg.72 #3-16

Functions

- A relation is any set of input that has an output
- A function is a relation where every input has exactly one output.
- Looking at a t-table, every x value must have exactly one y value.
- Looking at a graph, no vertical line can pass through two or more points.

Ex.1 Determine if the following is a function or relation.

function

a) $\{(3,2),(4,3),(5,4),(6,5)\}$

b)

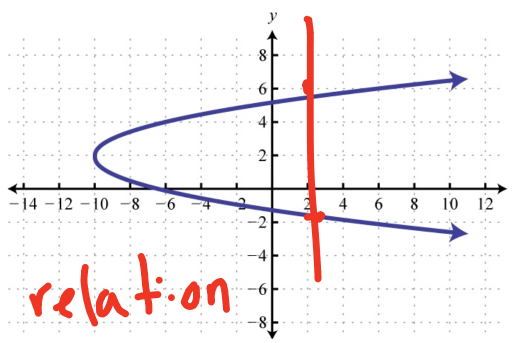
X	Y
-3	0
-2	-2
-1	5
0	-10
1	-4

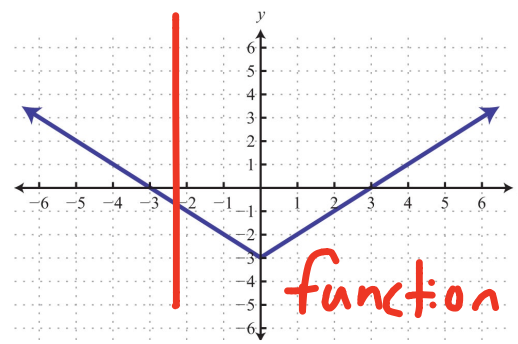
function

X	Y
1	-1
2	-2
3	-3
1	-3

relation

c)





Function form of an equation

- Function notation is a way to name a function. f(x) is pronounced f of x.

Evaluating Functions

- Substituting values for x.

Ex.1 Evaluate the function

a) $f(x) = 2x + 3$ when $x = -2$

$$f(-2) = 2(-2) + 3$$
$$= -4 + 3$$
$$= \boxed{-1}$$

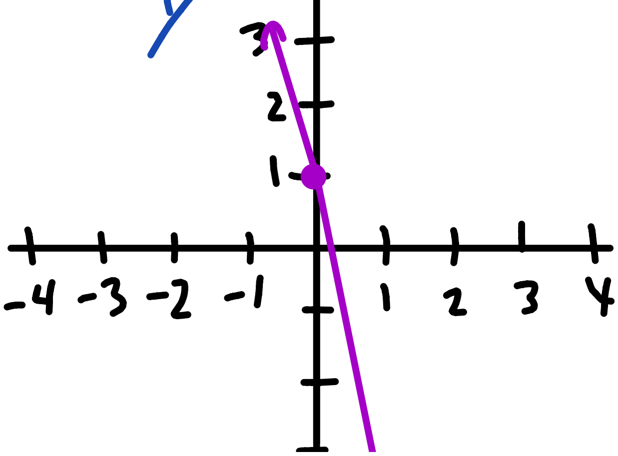
b) $f(x) = x^2 - 2x + 3$ when $x = -3$

$$f(-3) = (-3)^2 - 2(-3) + 3$$
$$= 9 + 6 + 3$$
$$= \boxed{18}$$

Graphing Linear Equations

Ex.1 by using a table: create a t-table.

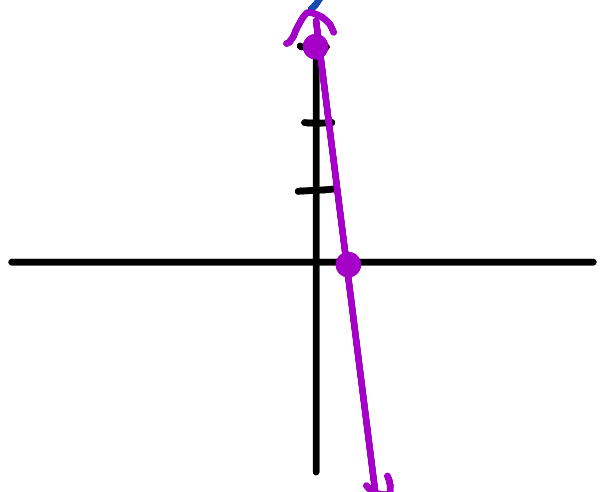
$y = -5x + 1$



x	y
0	1
1	-4
2	-9

Ex.2 by finding x and y intercepts. To find x intercepts, plug zero into y. To find y intercepts, plug zero into x.

$7x + y = 3$



x	y
(0,	3)
($\frac{3}{7}$,	0)

x	y
0	3
.43	0

Ex.3 Using slope intercept form. Must be in the form y = mx + b.

$m = \text{slope} = \frac{\text{rise}}{\text{run}}$

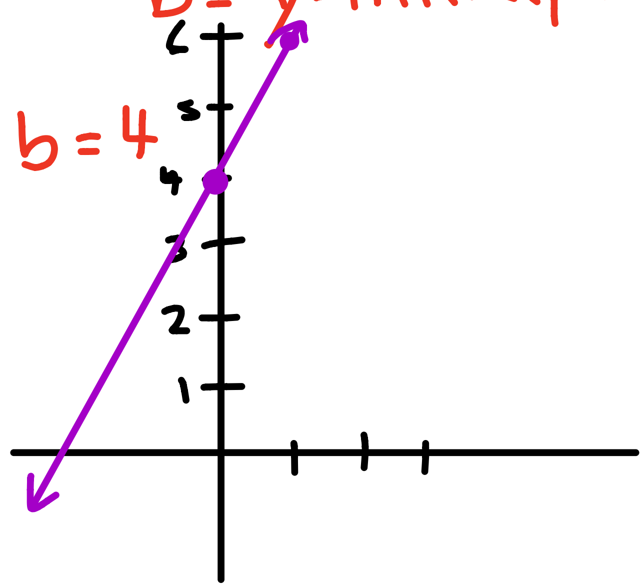
$b = \text{y-intercept}$

$-2x + y = 4$

$+2x \qquad +2x$

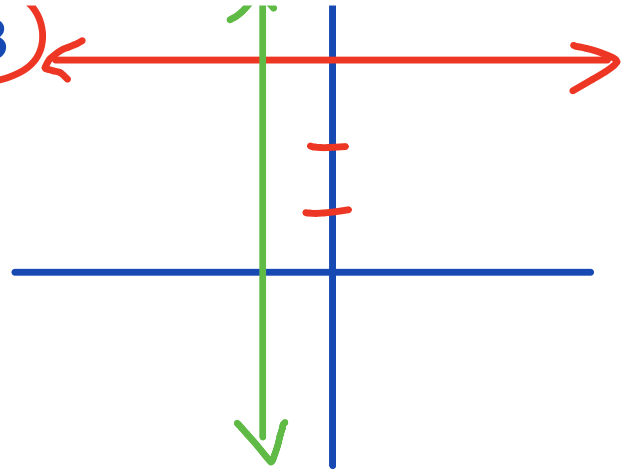
$y = 2x + 4$

$m = \frac{2}{1}$ $\quad b = 4$



Horizontal lines, $y = 3$

Vertical lines, $x = -1$



Rate of Change

- Rate of change is the ratio of change of one quantity to the change in another.
- Slope- is the rate of the vertical change (y) to the horizontal change (x).

$$\text{Slope} = \frac{\text{rise}}{\text{run}} = \frac{\text{change in } y}{\text{change in } x} = \frac{y_2 - y_1}{x_2 - x_1}$$

Ex.1 Find the slope $\overset{x_1\ y_1}{(2,4)}$ and $\overset{x_2\ y_2}{(4,8)}$.

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{8-4}{4-2} = \frac{4}{2} = \boxed{\frac{2}{1}}$$

Writing Linear Equations

- **Slope intercept form:**

$y = mx + b$

-

- **Point slope formula:**

$y - y_1 = m(x - x_1)$

Ex.1 Given slope and the y-intercept. Plug the point and slope into the slope intercept formula. m = -3/2, b = 7

$\frac{-3}{2}$

$y = mx + b$

$\boxed{y = \frac{-3}{2}x + 7}$

Ex.2 Given two points. Find the slope of the points, then plug them into the point slope formula. $(\overset{x_1}{2},\overset{y_1}{3})$, $(\overset{x_2}{-6},\overset{y_2}{7})$

$m = \frac{y_2 - y_1}{} = \frac{7-3}{} = \frac{4}{} = \frac{\boxed{-1}}{}$

$x_2 - x_1 \quad -6-2 \quad -8 \quad \boxed{2}$

$y - y_1 = m(x - x_1)$

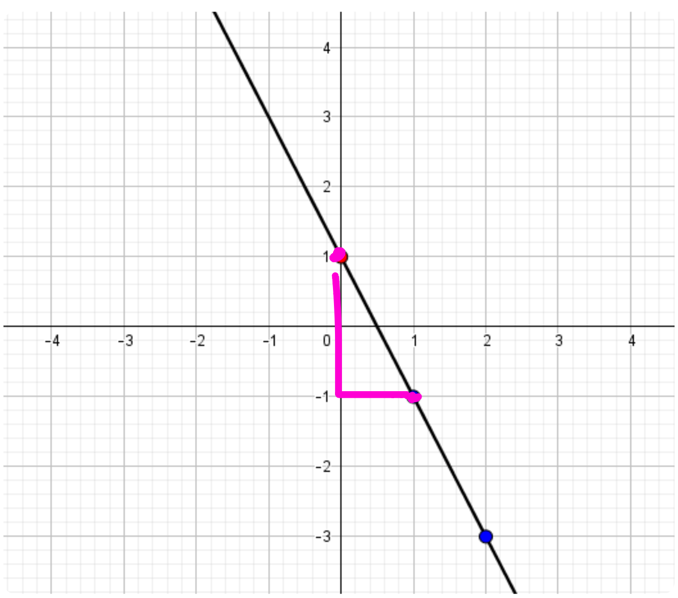
$y - 3 = -\frac{1}{2}(x - 2)$

$y - \cancel{3} = -\frac{1}{2}x + 1$

$\cancel{+3} \qquad +3$

$$\boxed{y = -\frac{1}{2}x + 4}$$

Ex.3 Given a graph. Find the y-intercept (b) and the slope (m), then plug them into the slope intercept formula.



$y = mx + b$

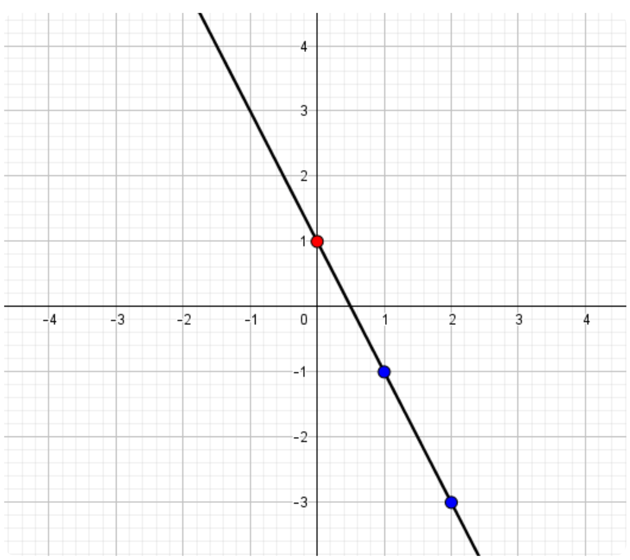
$b = 1$

$m = \frac{-2}{1}$

$$\boxed{y = \frac{-2}{1}x + 1}$$

characteristics of graphs

- Domain- all possible x values
- Range- all possible y values
- X intercept- where the graph crosses the x axis
- Y intercept- where the graph crosses the y axis.
- Interval of increase- domain of increase
- Interval of decrease- domain of decrease



Domain: $(-\infty,\infty)$

Range: $(-\infty,\infty)$

Interval Decrease: $(-\infty,\infty)$

X-intercept: $(.5,0)$

y-intercept $(0,1)$

end behavior

$x \to -\infty,\ y \to \infty$

$x \to \infty,\ y \to -\infty$

Systems of Equations by Graphing

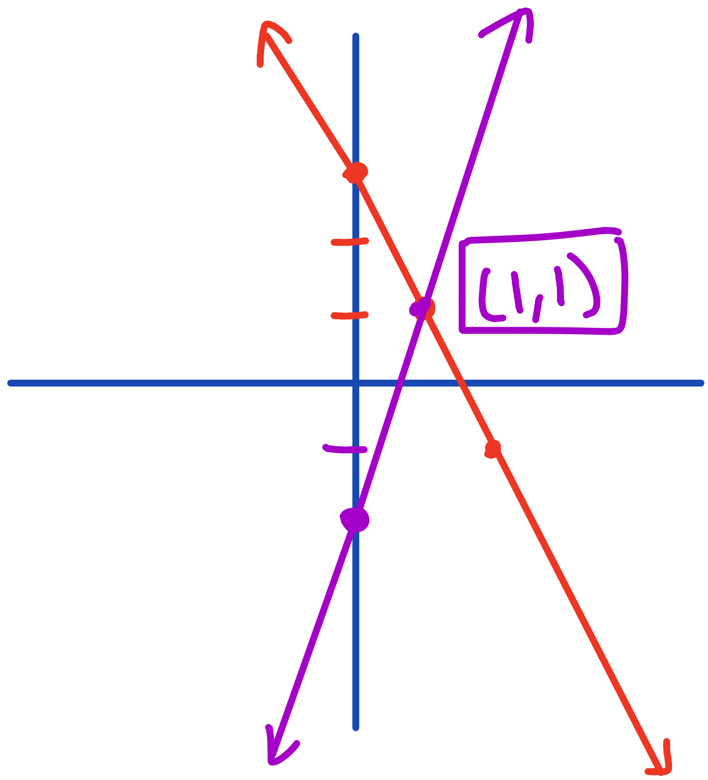
- Graph the equations, where they intersect are the solutions.
- If the graphs do not intersect, parallel lines, there are no solutions.
- If the equations are the same, there are infinitely many solutions.

Ex.1 one solution, lines intersect

$y + \cancel{2x} = 3$

$\cancel{-2x} \quad -2x$

$y = \frac{-2}{1}x + 3$

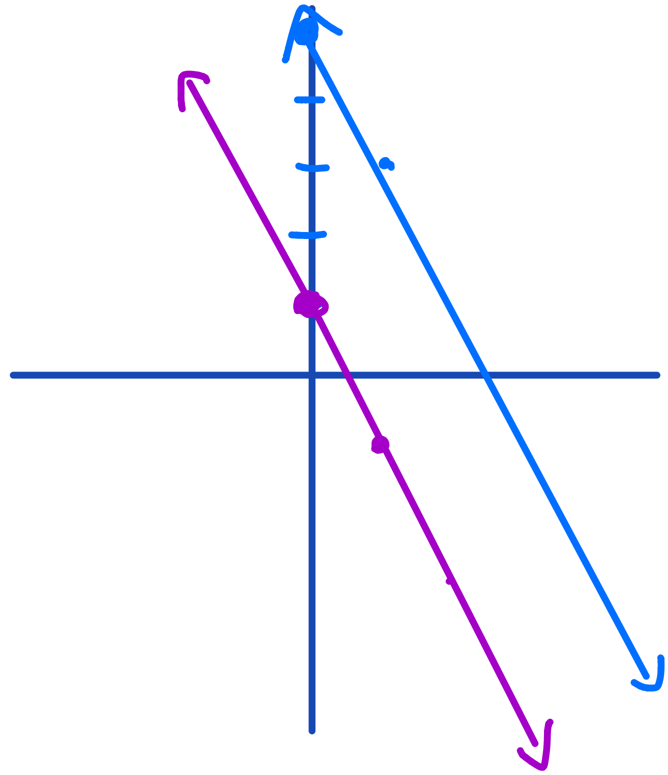


$y = \frac{3}{1}x - 2$

Ex.2 No solution, lines are parallel

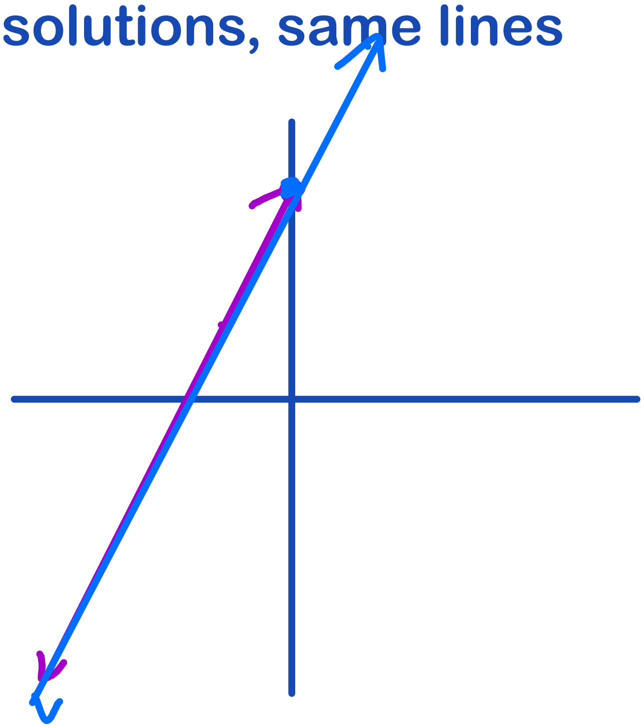
$$y = -2x + 5$$

$$2y - \cancel{2} = -4x$$
$$\quad\cancel{+2} \qquad +2$$
$$\frac{\cancel{2}y}{\cancel{2}} = \frac{-4x}{2} + \frac{2}{2}$$
$$\boxed{y = -2x + 1}$$



Ex.3 Infinitely many solutions, same lines

$$-2x = \cancel{3} - y$$
$$-3 \qquad \cancel{-3}$$
$$\frac{\cancel{-y}}{\cancel{-1}} = \frac{-2x}{-1} \frac{-3}{-1}$$
$$\boxed{y = 2x + 3}$$
$$-2x - 3 = -y$$
$$-y = -2x - 3$$
$$\boxed{y = 2x + 3}$$



By Elimination

1. write the equations with like terms in columns
2. Create opposite coefficients if needed.
3. Add the equations
4. Solve for the remaining variable

Ex.1 $2x - 2y = -8$

$2x + 2y = 4$

$\frac{\cancel{4}x}{\cancel{4}} = \frac{-4}{4}$

$x = -1$

$2(-1) + 2y = 4$

$\cancel{-2} + 2y = 4$

$\cancel{+2} \qquad +2$

$\frac{\cancel{2}y}{\cancel{2}} = \frac{6}{2}$

$y = 3$

$\boxed{(-1,3)}$

Ex.2 $\cancel{(2x-3y=4)2}$

$$-4x+5y=-8$$
$$4x-6y=8$$
$$-y=0$$
$$y=0$$

$$-4x+\cancel{5(0)}=-8$$
$$\frac{\cancel{-4}x}{\cancel{-4}}=\frac{-8}{-4}$$
$$x=2$$

$$\boxed{(2,0)}$$

Ex.3

$$(4x+5y=-2)4$$
$$(5x-4y=-23)5$$

$$16x+\cancel{20y}=-8$$
$$25x-\cancel{20y}=-115$$

$$\frac{\cancel{41}x}{\cancel{41}}=\frac{-123}{41}$$
$$x=-3$$

$$4(-3)+5y=-2$$
$$\cancel{-12}+5y=-2$$
$$\cancel{+12}\qquad +12$$
$$\frac{\cancel{5}y}{\cancel{5}}=\frac{10}{5}$$
$$y=2$$

$$\boxed{(-3,2)}$$

By substitution

1. solve for x and y.
2. Plug x or y into the other equation.
3. Solve for the variable.
4. Use the solution to find the other variable.

Ex.1 $X = -4$

$3x + 2y = 2$

$\boxed{(-4, 7)}$

$3(-4) + 2y = 2$

$-12 + 2y = 2$

$+12 \qquad +12$

$\frac{2y}{2} = \frac{14}{2}$

$\boxed{y = 7}$

Ex.2 $X + y = -1$ $\qquad x = -3$ $\qquad \boxed{(-3, 2)}$

$-y \quad -y$

$\boxed{x = -y - 1}$

$5x + y = -13$

$-y \quad -y$

$\frac{5x}{5} = \frac{-y - 13}{5}$

$X = \frac{-y - 13}{5}$

$5(-y - 1) = \left(\frac{-y - 13}{5}\right) \cancel{5}$

$-5y - 5 = -y - 13$

$+5y \; +13 \quad +5y \; +13$

$\frac{8}{4} = \frac{4y}{4}$

$\boxed{y = 2}$

Graphing Inequalities with Two Variables

1. Put Equation in slope intercept form.
2. Find the slope and y intercept, then graph.
   If < or >, use a dotted line
   If ≤ or ≥, use a solid line
3. Shade the region,
   If < or ≤, shade below.
   If > or ≥, shade above.

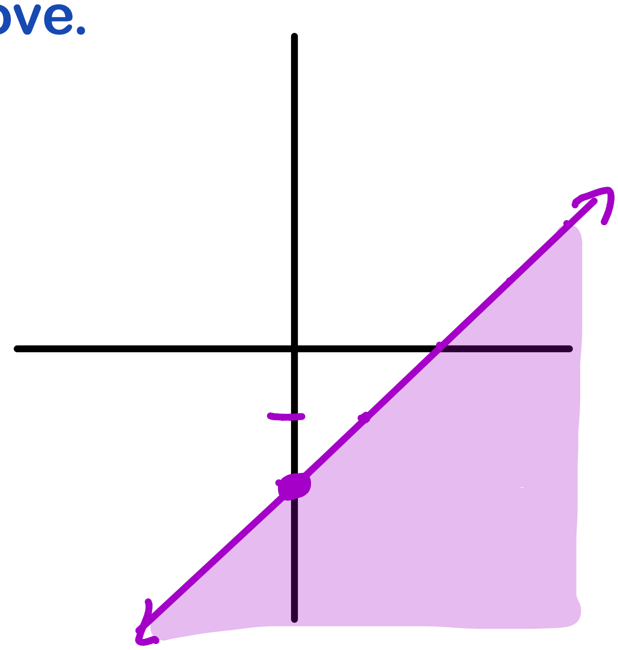
Ex.1 $\cancel{x} - y \geq 2$

$\cancel{-x} \qquad -x$

$\frac{\cancel{-}y}{\cancel{-1}} \geq \frac{-x}{-1} \frac{+2}{-1}$

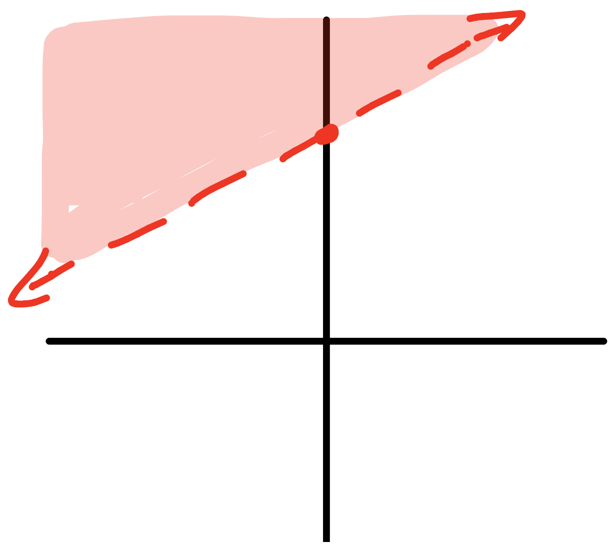
$y \leq x - 2$

$y = mx + b$



Ex.2 $\frac{\cancel{-4}y}{\cancel{-4}} > \frac{-2x}{-4} \frac{-12}{-4}$

$y > \frac{1}{2}x + 3$



Systems of Inequalities

1. Graph the two Inequalities
2. The shared shaded region is the solutions

Ex.1 $y > -x + 4$

$y < \frac{1}{2}x - 1$