Solving Equations

Properties of Equality

Addition Property of Equality: you can add the same number to both sides of the equation, and the statement remains true.

\[3 = 3\]
\[3 + 2 = 3 + 2\]
\[5 = 5\]
\[a = b\]
\[a + c = b + c\]

Subtraction Property of Equality: you can subtract the same number from both sides of the equation...

\[5 = 5\]
\[5 - 1 = 5 - 1\]
\[4 = 4\]
\[a = b\]
\[a - c = b - c\]

Multiplication Property of Equality: you can multiply both sides of the equation by the same number...

\[7 = 7\]
\[2(7) = 2(7)\]
\[14 = 14\]
\[a = b\]
\[c(a) = c(b)\]

Division Property of Equality: you can divide both sides of an equation by the same nonzero number...

\[\frac{8}{2} = \frac{8}{2}\]
\[\frac{a}{c} = \frac{b}{c}\] \[c \neq 0\]
**Distributive Property**  \( a(b+c) = ab + ac \)

**Ex. 1** Solve the equation using Properties of Equality.

a) \( 3x - 7 = 6 \)

b) \( \frac{1}{2} x + 4 = 10 \)

c) An ostrich that is 108 inches tall is 20 inches taller than 4 times the height of a kiwi in inches.

\( 108 \text{ in} = 4x + 20 \)

\( -20 \)

\( x = \frac{8}{3} \)

\( x = 2 \)

\( x = 12 \)
An emu that measures 60 inches in height is 70 inches less than 5 times the height of a kakapo. What is the height of a kakapo?

\[ 60 = 5x - 70 + 70 + 70 \]

\[ 130 = 5x \]

\[ x = \frac{130}{5} \]

\[ x = 26 \]

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Writing equations

Ex.1 One half an amount added to $50 is $262.

\[ \frac{1}{2} x + 50 = 262 \]

\[ -50 \quad -50 \]

\[ \frac{1}{2} x = 212 \cdot 2 \]

\[ x = 424 \]

Ex.2 Four times the sum of a number and 10 is 48.

\[ 4(x+10) = 48 \]

\[ 4x + 40 = 48 \]

\[ -40 \quad -40 \]

\[ x = 8 \]

\[ x = 2 \]

Ex.3 A rectangular garden is fenced on all sides with 256 feet of fencing. The garden is 8 feet longer than it is wide. Find the length and width of the garden.

\[ P = 256 \]

\[ w + 8 + w + 8 + w = 256 \]

\[ 4w + 16 = 256 \]

\[ -16 \quad -16 \]

\[ 4w = 240 \]

\[ \frac{4}{4} \]

\[ w = 60 \]
Ex. 4 One moving company charges $800 plus $16 per hour. Another company charges $720 plus $21 per hour. At what number of hours will the charge by both companies be the same?

Solving for a variable

Literal equations - are equations that contain two or more variables. Many literal equations are formulas.

Ex. 1 solve for the indicated variable.

a) \( V = \frac{L \cdot W \cdot h}{lw} \) 

\[ h = \frac{V}{lw} \]
b) \[ V(D) = \left( \frac{m}{V} \right)^{\lambda} \]

\[ m = VD \]

c) \[ Z(A) = \left( \frac{1}{2} \frac{(a+b)}{h} \right)^{\lambda} \]

\[ ZA = \frac{(a+b)h}{a+b} \]

\[ h = \frac{ZA}{a+b} \]

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Creating and Solving Inequalities

- when you multiply or divide by a negative, it changes the direction of the sign.

- is less than $\leq$
- is less than or equal to $\leq$
- is greater than $\geq$
- is greater than or equal to $\geq$
- is not equal to $\neq$

Ex. 1 solve the inequality

a) $2x \leq -2(4x+4)$

\[
\begin{align*}
2x &\leq -8x - 8 \\
2x + 8x &\leq -8 \\
10x &\leq -8 \\
\frac{10x}{10} &\leq \frac{-8}{10} \\
x &\leq -\frac{4}{5}
\end{align*}
\]

b) $\frac{1}{2}(-2x-12) > 4 - 6x$

\[
\begin{align*}
\frac{1}{2}(-2x-12) &> 4 - 6x \\
-\frac{1}{2}(-2x-12) &< 4 - 6x \\
-x - 12 &< 4 - 12x \\
-11x - 12 &< 4 \\
-11x &< 16 \\
x &> -\frac{16}{11}
\end{align*}
\]
Ex.2 writing inequalities.

a) It costs $20 to attend a play. A seasons pass cost $180. For what number of plays is it cheaper to pay $20 than to buy a seasons pass?

\[
\frac{20x}{20} < \frac{180}{20}
\]

\[x < 9\]

b) the sum of two consecutive integers is less than 83. Find the pair of integers with the greatest sum.
Creating and Solving Compound Statements

- Graphs of compound inequalities involving **AND** is the intersection of simple inequality graphs.
  \[ x > 2 \quad x < 6 \]

- Graphs of compound inequalities involving **OR** is the union of simple inequality graphs.
  \[ x \leq 2 \quad x > 6 \]

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Ex. 1 Solve each compound inequality

a) \( 4 \leq x + \frac{1}{2} \leq 8 \)
\[ \Rightarrow 2 \leq x \leq 6 \]
\[ x \leq 6 \]
\[ x \geq 2 \]

b) \(-5 \leq 2x + 3 \leq 9\)
\[ \Rightarrow -8 \leq 2x \leq 6 \]
\[ x \leq 3 \]
\[ x \geq -4 \]

c) \(-4 + x > 1\) OR \(-4 + x < -3\)
\[ x > 5 \]
\[ x < 1 \]

d) \(\frac{2x}{8} \leq \frac{6}{2}\) OR \(\frac{3x}{3} > 12\)
\[ x \leq 3 \]
\[ x > 4 \]
Ex. 2 write the compound inequality

a)\[ x \leq -1 \text{ or } x \geq 2 \]

b)\[ x \geq 0 \text{ and } x < 4 \]

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**Functions**

- A relation is any set of input that has an output.
- A function is a relation where every input has exactly one output.
- Looking at a t-table, every x value must have exactly one y value.
- Looking at a graph, no vertical line can pass through two or more points.
Ex. 1 Determine if the following is a function or relation.

a) \((2, 2), (4, 3), (5, 4), (6, 5)\)

- Function

b) | X  | Y  |
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>-3</td>
<td>0</td>
</tr>
<tr>
<td>-2</td>
<td>-2</td>
</tr>
<tr>
<td>-1</td>
<td>5</td>
</tr>
<tr>
<td>0</td>
<td>-10</td>
</tr>
<tr>
<td>1</td>
<td>-4</td>
</tr>
</tbody>
</table>

- Function

\[
\begin{array}{c|c|c|c}
X & Y & X & Y \\
-1 & \bullet & 1 & \bullet \\
3 & \bullet & 2 & \bullet \\
-3 & \bullet & 1 & \bullet \\
\end{array}
\]

- Relation

Function form of an equation
- Function notation is a way to name a function. \(f(x)\) is pronounced \(f\) of \(x\).

Evaluating Functions
- Substituting values for \(x\).
Ex. 1 Evaluate the function

a) \[ f(x) = 2x + 3 \quad \text{when} \quad x = -2 \]
\[ f(-2) = 2(-2) + 3 \]
\[ = -4 + 3 \]
\[ = 1 \]

b) \[ f(x) = x^2 - 2x + 3 \quad \text{when} \quad x = -3 \]
\[ f(-3) = (-3)^2 - 2(-3) + 3 \]
\[ = 9 + 6 + 3 \]
\[ = 18 \]

Graphing Linear Equations
Ex. 1 by using a table: create a t-table.

\[ y = -5x + 1 \]

<table>
<thead>
<tr>
<th>x</th>
<th>y</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>1</td>
<td>-4</td>
</tr>
<tr>
<td>2</td>
<td>-9</td>
</tr>
</tbody>
</table>
Ex. 2 by finding x and y intercepts. To find x intercepts, plug zero into y. To find y intercepts, plug zero into x.

\[ -7x + y = 3 \]

Ex. 3 Using slope intercept form. Must be in the form \( y = mx + b \).

\[-2x + y = 4\]
\[ y = 2x + 4 \]
\[ m = \frac{\text{rise}}{\text{run}} \]
\[ b = \text{y-intercept} \]

\[ m = \frac{2}{1} \]
\[ b = 4 \]
Rate of Change

- Rate of change is the ratio of change of one quantity to the change in another.
- Slope- is the rate of the vertical change (y) to the horizontal change (x).

\[
\text{Slope} = \frac{\text{rise}}{\text{run}} = \frac{\text{change in } y}{\text{change in } x} = \frac{y_2 - y_1}{x_2 - x_1}
\]

Ex. 1 Find the slope (2,4) and (4,8).

\[
m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{8 - 4}{4 - 2} = \frac{4}{2} = 2
\]
Writing Linear Equations

• Slope intercept form:
  \[ y = mx + b \]

• Point slope formula:
  \[ y - y_1 = m(x - x_1) \]

Ex.1 Given slope and the y-intercept. Plug the point and slope into the slope intercept formula. \( m = \frac{-3}{2}, \ b = 7 \)

Ex.2 Given two points. Find the slope of the points, then plug them into the point slope formula. \((2,3), (-6,7)\)

\[
m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{7 - 3}{-6 - 2} = \frac{4}{-8} = -\frac{1}{2}
\]
Ex. 3 Given a graph. Find the y-intercept \( b \) and the slope \( m \), then plug them into the slope intercept formula.

\[
\begin{align*}
\frac{x_2 - x_1}{-6 - 2} &= \frac{-8}{-12} \\
y - y_1 &= m(x - x_1) \\
y - 3 &= -\frac{1}{2}(x - 2) \\
&= \frac{-1}{2}x + 1 \\
y &= -\frac{1}{2}x + 4
\end{align*}
\]

\[
y = mx + b \\
b = 1 \\
m = -\frac{2}{1} \\
\Rightarrow y = -2x + 1
\]
characteristics of graphs

- Domain- all possible x values
- Range- all possible y values
- X intercept- where the graph crosses the x axis
- Y intercept- where the graph crosses the y axis.
- Interval of increase- domain of increase
- Interval of decrease- domain of decrease

end behavior

\[ x \to -\infty, \ y \to \infty \]
\[ x \to \infty, \ y \to -\infty \]
Systems of Equations by Graphing

- Graph the equations, where they intersect are the solutions.
- If the graphs do not intersect, parallel lines, there are no solutions.
- If the equations are the same, there are infinitely many solutions.

Ex. 1 one solution, lines intersect

\[ y + 2x = 3 \]
\[ -2x - 2x \]
\[ y = -\frac{2}{1}x + 3 \]
\[ y = 3x - 2 \]
Ex.2 No solution, lines are parallel

\[ y = -2x + 5 \]

\[ 2y - 2 = -4x \]
\[ 2y = -4x + 2 \]
\[ y = -2x + 1 \]

Ex.3 Infinitely many solutions, same lines

\[ -3x = 3 - y \]
\[ -3 \]
\[ y = -\frac{2x - 3}{-1} \]
\[ y = 2x + 3 \]
\[ -2x - 3 = -y \]
\[ y = 2x + 3 \]
By Elimination

1. write the equations with like terms in columns
2. Create opposite coefficients if needed.
3. Add the equations
4. Solve for the remaining variable

Ex.1 \[2x - 2y = -8\]
\[2x + 2y = 4\]

\[\frac{4x}{4} = -\frac{4}{4}\]
\[x = -1\]

\[2(-1) + 2y = 4\]
\[-2 + 2y = 4\]
\[+2\]
\[y = 3\]

\[(1, 3)\]
Ex. 2
\[
\begin{align*}
2x - 3y &= 11 \\
-4x + 5y &= -8 \\
4x - 6y &= 8 \\
\end{align*}
\]
\[
\begin{align*}
-y &= 0 \\
y &= 0 \\
\end{align*}
\]
\([
(2, 0)
]\)

Ex. 3
\[
\left\{
\begin{align*}
4x + 5y &= -2 \\
5x - 4y &= -23 \\
\end{align*}
\right.
\]
\[
\begin{align*}
16x + 20y &= -8 \\
25x - 20y &= -115 \\
\end{align*}
\]
\[
\begin{align*}
4x &= -123 \\
x &= -3 \\
\end{align*}
\]
\[
\begin{align*}
5y &= 10 \\
y &= 2 \\
\end{align*}
\]
\([
(-3, 2)
]\)
By substitution

1. solve for x and y.
2. Plug x or y into the other equation.
3. Solve for the variable.
4. Use the solution to find the other variable.

Ex. 1 \[ x = -4 \]
\[ 3x + 2y = 2 \]
\[ 3(-4) + 2y = 2 \]
\[ -12 + 2y = 2 \]
\[ 2y = 14 \]
\[ y = 7 \]

Ex. 2 \[ x + y = -1 \]
\[ 5x + y = -13 \]
\[ 5(-y - 1) = -y - 13 \]
\[ -5y - 5 = -y - 13 \]
\[ +5y + 13 + 5y + 13 \]
\[ y = 2 \]
Graphing Inequalities with Two Variables

1. Put Equation in slope intercept form.
2. Find the slope and y intercept, then graph.
   - If < or >, use a dotted line
   - If ≤ or ≥, use a solid line
3. Shade the region,
   - If < or ≤, shade below.
   - If > or ≥, shade above.

Ex.1 \( x - y \geq 2 \)
\[
\begin{align*}
-y & \geq -x + 2 \\
y & \leq x - 2 \\
y &= mx + b
\end{align*}
\]

Ex.2 \( \frac{4y}{6} > \frac{-2x - 12}{4} \)
\[
\begin{align*}
-4y & > -2x - 12 \\
y & > \frac{1}{2}x + 3
\end{align*}
\]
Systems of Inequalities

1. Graph the two inequalities
2. The shared shaded region is the solutions

Ex. 1 \[ y > -x + 4 \]
\[ y < \frac{1}{2} x - 1 \]