Greatest common factor

-The highest number or variable that divides exactly into two or more terms

Standard form:

$y = ax^2 + bx + c$

Intercept form:

$y = a(x-p)(x-q)$

Ex.1 find the GCF

(A) 12 and 15

3 | 12 15

4 5

3

(B) x^2y^4 and x^3

x^2 | x^2y^4 x^3

1 4 x

x^2

Ex.2 what is the GCF

A) $3x-6$

3

$3 \mid 3x \quad -6$

$x \quad -2$

B) $5x^3-15x^2$

$5x^2$

$5 \mid 5x^3 \quad -15x^2$

$x^2 \mid x^3 \quad -3x^2$

$x \quad -3$

Ex.3 Factor

A) $3x-6$

$3 \mid 3x \quad -6$

$x \quad -2$

$3(x-2)$

$3x-6$

B) $5x^3-15x^2$

$5x^2 \mid 5x^3 \quad -15x^2$

$x \quad -3$

$5x^2(x-3)$

Factoring quadratics

1. Put the quadratic in standard form. $ax^2 + x + c$
2. Factor out GCF; make 'a' positive.
3. Multiply 'a' times 'c'.
4. Write out factors of 'ac'
5. Pick the factors that add to be 'b'.

Ex.1 Factor the quadratic when a=1

a=1 b=8 c=7

A b^2+8b+7

$(b+1)(b+7)$

a·c
1·7
7
1·7
-1·-7

$(b+1)(b+7)$

$b^2+7b+b+7$

b^2+8b+7

B $k^2-13k+40$

a=1 b=-13 c=40

$(k-5)(k-8)$

a·c
1·40
40
-1·-40
-2·-20
-4·-10
-5·-8

Ex.2 Factor the quadratic when a>1

$a=5$ $b=19$ $c=12$

A $5n^2+19n+12$

$(5n+6)\left(\frac{5n}{5}+\frac{10}{5}\right)$

$(5n+6)(n+2)$

$a \cdot c$
$5 \cdot 12$
60
$1 \cdot 60$
$2 \cdot 30$
$3 \cdot 20$
$4 \cdot 15$
$5 \cdot 12$
$6 \cdot 10$

B $2n^3+3n^2-9n$

$n(2n^2+3n-9)$

$n(2n-3)(2n+6)$

$n(2n-3)(n+3)$

$a \cdot c$
$2 \cdot -9$
-18
$1 \cdot 18$
$2 \cdot 9$
$-3 \cdot 6$

Ex.3 Factor the quadratic when there is no b.

ax^2+bx+c

A $9x^2-1$

$a=9 \quad b=0 \quad c=-1$

$a \cdot c$
-9
$1 \cdot 9$
$-3 \cdot 3$

$\left(\frac{9x}{3}-\frac{3}{3}\right)\left(\frac{9x}{3}+\frac{3}{3}\right)$

$(3x-1)(3x+1)$

Ex.4 Factor the quadratic when there is no c.

A $\frac{3x^2}{3x}+\frac{12x}{3x}$

$3x(x+4)$

Solving quadratic's

Solving by factoring

1. Factor the quadratic
2. Set factors equal to zero
3. Solve for X

Ex.1 Solve by factoring

A $n^2-10n+22=-2$

$+2 \quad +2$

$n^2-10n+24=0$

$a=1 \quad b=-10 \quad c=24$

$(n-4)(n-6)=0$

$n-4=0$

$+4 \quad +4$

$n=4$

$n-6=0$

$+6 \quad +6$

$n=6$

$a \cdot c$

$1 \cdot 24$

24

$1 \cdot 24$

$2 \cdot 12$

$3 \cdot 8$

$-4 \cdot -6$

B $6n^2 - 18n - 18 = 6$
$-6 \quad -6$

$\frac{6n^2}{6} - \frac{18n}{6} - \frac{24}{6} = \frac{0}{6}$

$n^2 - 3n - 4 = 0$

$a=1$ $b=-3$ $c=-4$

$(n+1)(n-4) = 0$

$n+1=0$ $n-4=0$

$-1 \quad -1 \quad +4 \quad +4$

$n=-1$ $n=4$

a·c
1·-4
-4
1 -4
2 2

Solve by taking the square root

*you can undo a square by taking the square root

Ex.1 one step

A $\sqrt{x^2} = \sqrt{25}$

$x = \pm 5$ $x=5$ $x=-5$

B $\sqrt{x^2} = \sqrt{32}$

$x = \pm\sqrt{32}$

$\sqrt{16}\sqrt{2}$

$x = \pm 4\sqrt{2}$

C) $3x^2 - 16x = 0$

Ex.2 two step

A $x^2 - 7 = 9$

$+7 \quad +7$

$\sqrt{x^2} = \sqrt{16}$

$x = \pm 4$

B $\frac{2x^2}{2} = \frac{14}{2}$

$\sqrt{x^2} = \sqrt{7}$

$x = \pm\sqrt{7}$

$x = \pm 2.6$

Ex.3 three step

A $(x+3)^2 - 7 = 2$

$+7 \quad +7$

$\sqrt{(x+3)^2} = \sqrt{9}$

$x + 3 = \pm 3$

$-3 \quad -3$

$x = \pm 3 - 3$

$x = 3 - 3$

$x = 0$

$x = -3 - 3$

$x = -6$

B $3(x+3)^2 - 12 = 0$

$+12 \quad +12$

$\frac{3(x+3)^2}{3} = \frac{12}{3}$

$\sqrt{(x+3)^2} = \sqrt{4}$

$x+3 = \pm 2$

$-3 \quad -3$

$x = \pm 2 - 3$

$x = -1$

$x = -5$

Completing the square

1. Put quadratic in standard form ax^2+bx+c
2. Move 'c' to the other side of the equation $ax^2+bx\_\_ = -c$
3. Make 'a' one
4. Find the new 'c'. $c = \left(\frac{b}{2}\right)^2$
5. Add new 'c' to both sides
6. Factor left side of equation
7. Solve for X

Ex.1 Solve by completing the square.

A $p^2+16p-22=0$

$+22 \quad +22$

$p^2+16p+(8)^2 = 22+(8)^2$

$p^2+16p+64 = 22+64$

$(p+8)(p+8) = 86$

$\sqrt{(p+8)^2} = \sqrt{86}$

$p+8 = \pm\sqrt{86}$

$-8 \quad -8$

$p = -8 \pm \sqrt{86}$

a·c
64
1·64
2·32
4·16
8·8

B) $\frac{2n^2}{2} + \frac{12n}{2} + \frac{10}{2} = \frac{0}{2}$

$n^2 + 6n + 5 = 0$

$-5 \quad -5$

$n^2 + 6n + (3)^2 = -5 + (3)^2$

$\sqrt{(n+3)^2} = \sqrt{4}$

$n + 3 = \pm 2$

$-3 \quad -3$

$n = -3 \pm 2$

$n = -1$ $\quad$ $n = -5$

C $m^2 - 12m + 26 = 0$

$-26 \quad -26$

$m^2 - 12m + (-6)^2 = -26 + (-6)^2$

$\sqrt{(m-6)^2} = \sqrt{10}$

$m - 6 = \pm\sqrt{10}$

$+6 \quad +6$

$m = 6 \pm \sqrt{10}$

Quadratic formula

1. put quadratic in standard form $ax^2 + bx + c$
2. Find 'a', 'b', and 'c' and plug them into the quadratic formula.

$$x = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$$

Ex. 1 $2x^2 - 3x = 5$

$-5 \quad -5$

$a=2 \quad b=-3 \quad c=-5$

$2x^2 - 3x - 5 = 0$

$$x = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$$

$$x = \frac{+3 \pm \sqrt{(-3)^2-4(2)(-5)}}{2(2)}$$

$$x = \frac{3 \pm \sqrt{9+40}}{4}$$

$$x = \frac{3 \pm \sqrt{49}}{4}$$

$$x = \frac{3 \pm 7}{}$$

$$x = \frac{10}{4} = \boxed{\frac{5}{2} = 2.5} \qquad x = \frac{-4}{4} = \boxed{-1}$$

Ex2 $2m^2 - 7m - 3 = 0$

$a = 2 \quad b = -7 \quad c = -3$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-(-7) \pm \sqrt{(-7)^2 - 4(2)(-3)}}{2(2)}$$

$$x = \frac{7 \pm \sqrt{49 + 24}}{4}$$

$$\boxed{x = \frac{7 \pm \sqrt{73}}{4}}$$

$$x = \frac{7 + \sqrt{73}}{4} \qquad x = \frac{7 - \sqrt{73}}{4}$$

Quadratic word problems

Ex. 1 The area of a square is 40 square centimeters. What is the length of one side of the square?

$$A = s^2 \qquad \sqrt{40} = \sqrt{s^2}$$

$$s = \pm\sqrt{40} \qquad \boxed{s = 2\sqrt{10}}$$

Ex.2 The area of a circle is 60 square millimeters. What is the radius of the circle?

$A = \pi r^2$

$\frac{60}{\pi} = \frac{\pi r^2}{\pi}$

$\sqrt{r^2} = \sqrt{19.1}$

$r = \sqrt{19.1}$