CRM Polynomial Expressions Name: ______________________ Period: __

Algebraic Expressions

An expression containing variables, numbers, and operation symbols is called an ______________________ ________________________.
An expression does NOT contain an equal sign. An example of an algebraic expression is 5x + 7y – 3.

In an **algebraic expression**, there are four different parts: coefficients, variables, constants, and terms. 5x + 7y - 3

Variables are the letters in an expression.

Coefficients are the numbers in front of the variables.

Constants are the "plain numbers" or terms **Terms** are separated by a + or – sign and can be numbers without variables. and/or variables.

Classifying and Adding/Subtracting Polynomials

Polynomials

<table>
<tr>
<td>Definition
A ________________ is an expression that can have constants, variables, and exponents. Polynomials CANNOT contain
• Radicals
• Fraction exponents
• Negative exponents
• Variables in the denominator</td>
<td>Characteristics
Polynomials are named by their ___________ and number of ____________.
• The degree is the ___________ exponent of a variable.
• Example: What is the degree of the following?
a) $2x^2 + 5x - 3$ b) $4x - 3x^5 + 2x^2 - 1$</td>
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<tr>
<td colspan="2">Which of the following are examples of polynomials?
$3x^4 - 7$ $\sqrt{x} + 2$ $\frac{x+1}{x^3}$ $6x^{-2} - 3x$ $5x$ $\frac{1}{3}x^2 + 4x - 9$</td>
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</table>

- **Standard Form –** the terms are arranged in _____________ order from the _____________ exponent to the _____________ exponent.
- **Degree** – the ________________ exponent of the variable in the polynomial.

Rewrite each polynomial in standard form. Then identify the degree of the polynomial:

	$5x - 6x^2 - 4$	$-7x + 8x^2 - 2 - 8x^2$	$6(x-1) - 4(3x^2) - x^2$
Standard form:			
Degree:			

Classifying Polynomials

Polynomial are classified by degree and number of terms:

Degree	Name	Example
0		
1		
2		
3		
4+		

Terms	Name	Example
1		
2		
3		
4+		

Complete the table below. Simplify the expressions or put in standard form if necessary.

Polynomial	Degree	# of Terms	Classification
$8x$			
10			
$-24 + 3x - x^2$			
$7x - 9x + 1$			
$4x^2 - 5x^3 - 4 + 5x - 1$			
$2x + 3 - 7x^2 + 4x + 7x^2$			

Combining Like Terms

Terms with the same variable raised to the same exponent are like terms.

Like: $3x$ *and* $-7x$	**Like:** $2y^2$ *and* $6y^2$	**Not Like:** $4x$ *and* $6x^2$

Directions: Simplify the following expressions:

1. $-3x + 6x$

2. $y - 3 + 6 - 2y$

3. $8m + n - 3 + 10$

4. $9x - 10x^2 + 7x - 3$

Multiplying Variables

$a^b \cdot a^c = a^{b+c}$

When multiplying expressions with the same bases, __________________ the exponents.

1. $x^2 \cdot x^5$

2. $a^3 \cdot a^4$

3. $y^2 \cdot y^5 \cdot z^2$

Distributive Property

$a(b + c) = ab + ac$

1. $5(x + 2)$	2. $-3(x - 4)$	3. $-6(-2x - 3)$
4. $4x - 5(x - 1)$	5. $-2(4 + x) + 4(2 - 8x) + 5$	6. $2(3 + x) + 4(1 - 4x) + 5$

Evaluating Expressions

When you evaluate an expression, you are replacing the variable with what the variable equals.

Evaluate: $4x - 5$ when $x = 6$

Practice: Evaluate the following expressions if $m = 7, r = 8,$ *and* $t = -2$

1. $5m - 6$	2. $\frac{r}{t}$	3. $3m - 5t$	4. $t^2 - 4r$

Adding Polynomials

When adding, use the following steps to add polynomials:

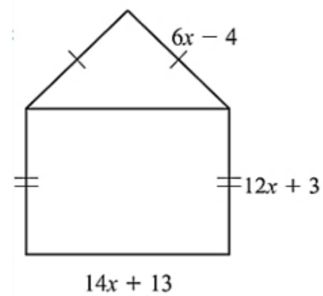
- Get rid of the parentheses first!
- Identify and combine like terms
- Make sure final answer is in standard form

1. $(4x^2 + 2x + 8) + (8x^2 + 3x + 1)$

2. $(-2x + 5) + (-4x^2 + 6x + 9)$

Application: Find an expression that represents the perimeter of the house.

What does it mean to find the perimeter of an object?



Perimeter of the house:

Subtracting Polynomials

Subtracting polynomials is like adding polynomials except we must take care of the minus sign first. Subtracting polynomials require the following steps:

- Change the sign of the terms in the parentheses after the subtraction sign
- Identify and combine like terms
- Add (Make sure final answer is in standard form)

1. $(7x^2 - 2x + 1) - (3x^2 + 4x - 7)$

2. $(3x^2 + 5x) - (4x^2 + 7x - 1)$

3. $(5x^3 - 4x + 8) - (-2 + 3x)$

4. $(3 - 5x + 3x^2) - (-x + 2x^2 - 4)$

Multiplying Polynomials

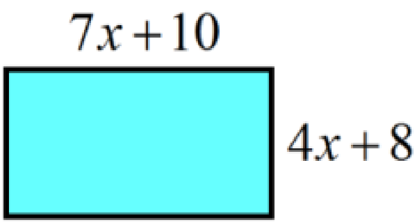
1. $4x(x+3)$	2. $(x-3)(x+7)$	3. $(x+5)^2$
4. $(x-4)(x+4)$	5. $(3x+6)(2x-7)$	6. $((x-3)(2x^2+2)$

Practice

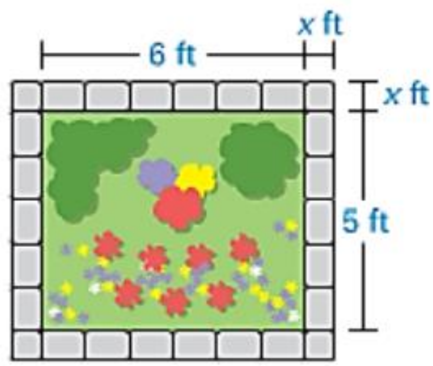
1. $(x-7)(x+4)$
2. $(x-9)^2$
3. $(x+10)(x-10)$
4. $x(x-12)$
5. $(3x+7)(2x+1)$
6. $(4x-5)(3x-6)$

Applications Using Polynomials

1. Write an expression that represents the area and perimeter of this rectangle.



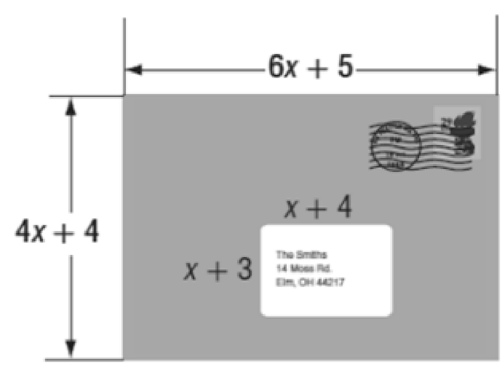
2. You are designing a rectangular flower bed that you will border using brick pavers. The width of the board around the bed will be the same on every side, as shown.



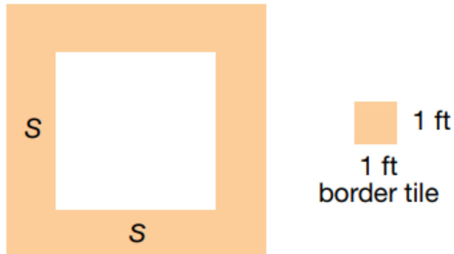
a. Write a polynomial that represents the total area of the flower bed and border.

b. Find the total area of the flower bed and border when the width of the border is 1.5 feet.

3. Find the expression that represents the area not covered by the mailing label.



Task 5: Swimming Pool You want to build a square swimming pool in your backyard. Let s denote the length of each side of the swimming pool (measured in feet). You plan to surround the pool by square border tiles, each of which is 1 foot by 1 foot (see figure).



A teacher asks her students to find an expression for the number of tiles needed to surround such a square pool, and sees the following responses from her students:

$4(s+1)$
s^2
$4s+4$
$2s+2(s+2)$
$4s$

Is each mathematical model correct or incorrect? How do you know?

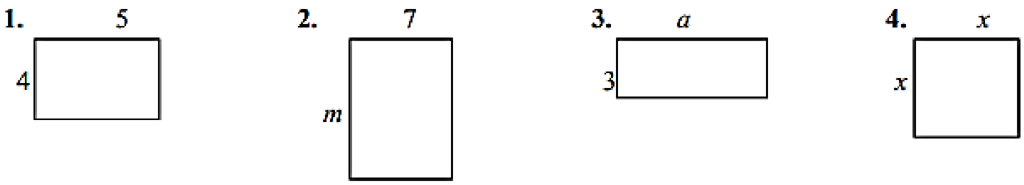
Task #9: Expression Pairs: Equivalent or Not?

$a+(3-b) \; and \; (a+3)-b$	$\frac{1}{x+y} \; and \frac{1}{x}+\frac{1}{y}$
$2+\frac{k}{5} \; and \; 10+k$	$\sqrt{(x^2+y^2)} \; and \; x+y$
$(a-b)^2 \; and \; a^2 \; -b^2$	$bc-cd \; andc(b-d)$
$3(z+w) \; and \; 3z+3w$	$(2x)^2 and \; 4x^2$
$-a+2 \; and \; -(a+2)$	$2x+4 \; and \; x+2$
$x^2+4x^2 and \; 5x^2$	

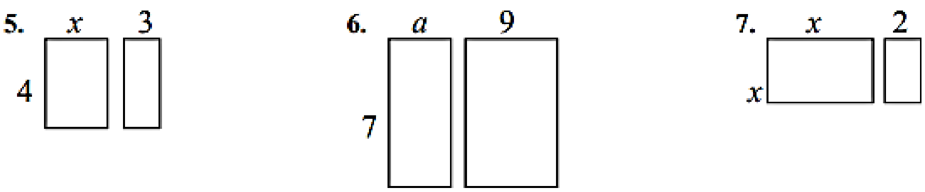
Distributive Property Using Area

NAME ______________________

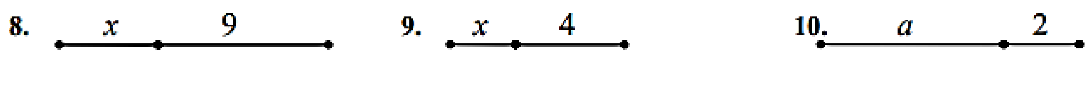
Write the expression that represents the area of each rectangle.



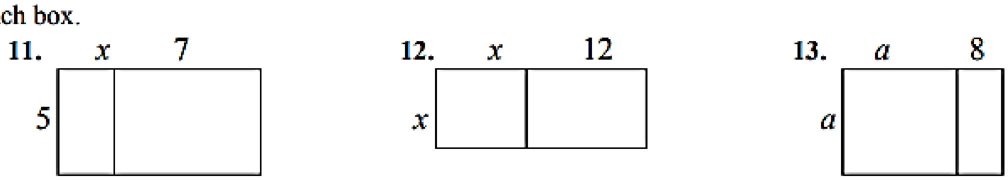
Find the area of each box in the pair.



Write the expression that represents the total length of each segment.



Write the area of each rectangle as the product of *length* × *width* and also as a sum of the areas of each box.



AREA AS PRODUCT	AREA AS SUM
5(x+7)	5x+35

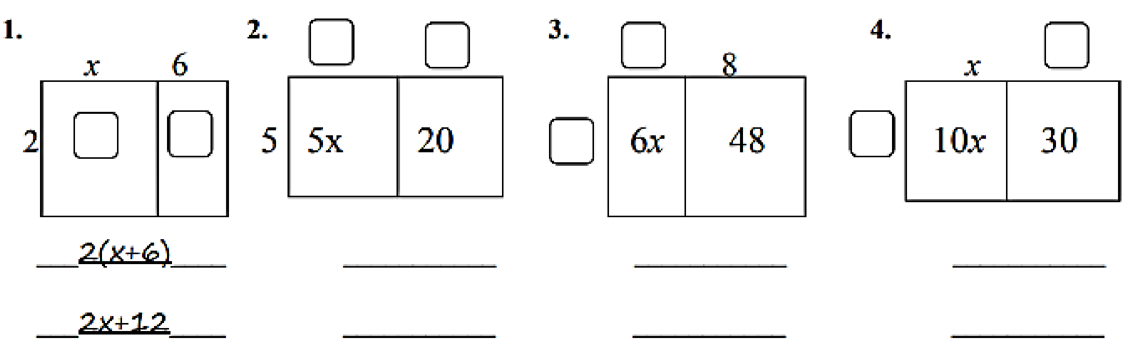
AREA AS PRODUCT	AREA AS SUM

AREA AS PRODUCT	AREA AS SUM

Factoring a Common Factor Using Area

NAME ______________________

Fill in the missing information for each: dimensions, area as product, and area as sum



Creating and Translating Algebraic Expressions

The Commutative and Associative Properties

Commutative Property of Addition
(order doesn't matter)

$$5 + 6 \text{ can be written as } 6 + 5$$

Associative Property of Addition
(grouping order doesn't matter)

$$2 + (5 + 6) \text{ can be written as } (2 + 6) + 5$$

Commutative Property of Multiplication
(order doesn't matter)

$$5 \cdot 6 \text{ can be written as } 6 \cdot 5$$

Associative Property of Multiplication
(grouping order doesn't matter)

$$(2 \cdot 5) \cdot 6 \text{ can be written as } 2 \cdot (6 \cdot 5)$$

Addition	Subtraction	Multiplication	Division	Exponents
Sum	Difference	Of	Quotient	Power
Increased by	Decreased by	Product	Ratio of	Squared
More than	Minus	Times	Each	Cubed
Combined	Less	Multiplied by	Fraction of	
Together	Less than	Double, Triple…	Out of	
Total of	Fewer than	Twice	Per	
Added to	How many more	As much	Divided by	
Gained	Left	Each	Split	
Plus				

Write the following as expressions.

Addition
The sum of x and 4.

Multiplication
The product of x and 3.

Subtraction
The difference of x and 5.

X decreased by 5.

Five less than x.

Division
The quotient of x and 7.

The ratio of x and 7.

Practice: Write the expression for each verbal description.

1. The difference of a number and 5	2. The quotient of 14 and 7	3. y decreased by 17
4. x increased by 6	5. The sum of a number and 8	6. 6 squared
7. Twice a number	8. 8 more than a third of a number	9. 6 less than twice k
10. Five divided by the sum of a and b	11. The quotient of k decreased by 4 and 9.	12. 2 minus the quantity 3 more than p
13. Half of the quantity 1 less than w.	14. Nine less than the total of a number and 2.	15. The product of a number and 3 decreased by 5.

Practice: Write each as a verbal expression.

16. $\frac{x}{2}$	17. $a + 9$
18. $5n - 7$	18. $3(y + 7)$

Your Birthday!
Here's a fun trick to show a friend, a group, or an entire class of people. Tell the person (or class) to think of their birthday and you will guess it.

Step 1) Have them take the month number from their birthday: January = 1, Feb = 2, etc.
Step 2) Multiply that by 5.
Step 3) Then add 6.
Step 4) Then multiply that total by 4.
Step 5) Then add 9.
Step 6) Then multiply this total by 5 once again.
Step 7) Finally, have them add to that total the day in which they were born.
Step 8) Subtract 165

Magic Math: Birthday Trick
Do you believe that I can figure out your birthday by using simple math? Get a calculator and ask your classmate to try the following. Your classmate must press equal (or enter) between every step.

a) Enter the month of his/her birth into the calculator. (Ex: enter 5 for May)
b) Multiply that number by 7.
c) Subtract 1 from that result.
d) Multiply that result by 13.
e) Add the day of birth. (Ex: For June 14th add 14)
f) Add 3.
g) Multiply by 11.
h) Subtract the month of birth.
i) Subtract the day of birth.
j) Divide by 10.
k) Add11.
l) Divide by 100.

Matching

I have **-4x** **Who has** **half of the difference of four times a number and eight**	**I have** **2x-4** **Who has** **a third of the difference of eighteen and six times a number**
I have **6-2x** **Who has** **three multiplied by the result of three subtracted from ten times a number**	**I have** **30x-9** **Who has** **the difference of a number and seven**
I have **x-7** **Who has** **six subtracted from the opposite of the sum of three times a number and five**	**I have** **-3x-11** **Who has** **the difference of seven times a number and one**
I have **7x-1** **Who has** **the difference of eight times a number and four times the same number**	**I have** **4x** **Who has** **eight more than three times the sum of a number and one**

Matching.

I have **3x+11** **Who has** **two times a number plus four**	**I have** **2x+4** **Who has** **add ten to three times a number subtracted from one**
I have **-3x+11** **Who has** **the difference of three times a number and eleven**	**I have** **3x-11** **Who has** **half the sum of double a number and fourteen**
I have **x+7** **Who has** **subtract ten from a number and multiply the result by three**	**I have** **3x-30** **Who has** **the difference of twice a number and six**
I have **2x-6** **Who has** **seven times a number subtracted from the same number**	**I have** **-6x** **Who has** **the difference of four times a number and eight times the same number**

Introduction to Sequences

-A sequence is simply an ordered list of numbers.
-Each number in the sequence is called a "term."
-Terms are referred to by the following notation:
If we refer to a generic term of the sequence, we say a_n.

5,	7,	9,	11,	13,	15,...
↑	↑	↑	↑	↑	↑
a_1	a_2	a_3	a_4	a_5	a_6 ...

Arithmetic Sequences

Arithmetic Sequences are built by repeatedly adding the same number (called the common difference) to the first term a1.

Arithmetic: 17, 13, 9, 5, 1, -3, -7…

$a_1 = 17$

common difference =

Geometric Sequences

Geometric Sequences are built by repeatedly multiplying the same number (called the common ratio) to the first term a1.

Geometric: ¾, 3, 12, 48, 192, …

$a_1 = \frac{3}{4}$

common ratio =

Square Sequences

$x_n = n^2$

Square: 1, 4, 9, 16, 25…

Cube Sequences

$x_n = n^3$

Cube: 1, 8, 27, 64, 125…

Fibonacci Sequence

The next number is found by adding the two previous numbers

0, 1, 1, 2, 3, 5, 8, 13, 21, 34, ...

Practice: Find the next term.

1. $-4, -2, 0, 2, \ldots$

2. $9, 4, -1, \ldots$

3. $\frac{1}{2}, \frac{3}{4}, 1, \frac{5}{4}, \ldots$

4. $\frac{1}{4}, \frac{3}{4}, \frac{5}{4}, \frac{7}{4}, \ldots$

5. 1, 3, 9, 27, 81, …

6. 200, 100, 50, 25, …

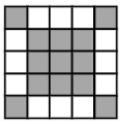
7. $-2, 1, -\frac{1}{2}, \frac{1}{4}, \ldots$

8. 2, 5, 10, 17, 26,…

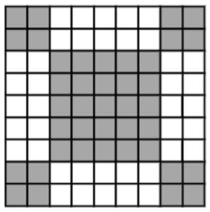
CRM Task 8

In Prague some sidewalks are made of small square blocks of stone. The blocks are in different shades to make patterns that are in various sizes.

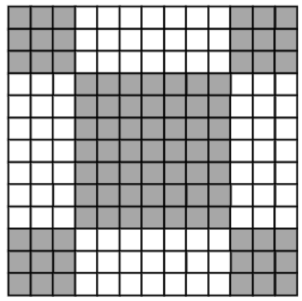
Draw the next pattern in this series.



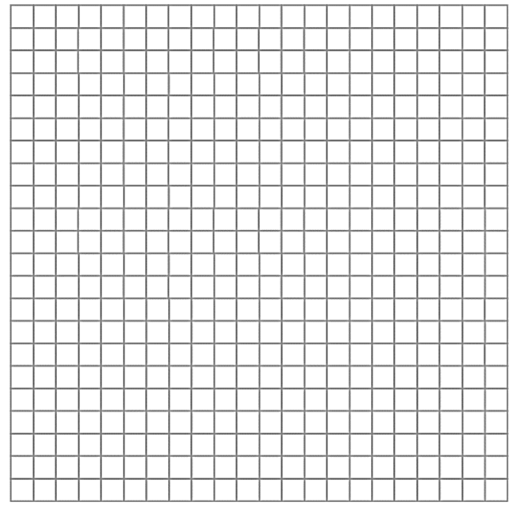
Pattern #1



Pattern #2



Pattern #3



Pattern #4

1. Complete the table below

Pattern number, n	1	2	3	4
Number of white blocks	12	40		
Number of gray blocks	13			
Total number of blocks	25			

2. What do you notice about the number of white blocks and the number of gray blocks?

____

3. The total number of blocks can be found by squaring the number of blocks along one side of the pattern.

a. Fill in the blank spaces in this list. $25 = 5^2$ 81= ____ 169 = ____ $289 = 17^2$

b. How many blocks will pattern #5 need? ____

c. How many blocks will pattern #n need? ____

4. a. If you know the total number of blocks in a pattern you can work out the number of white blocks in it. Explain how you can do this.

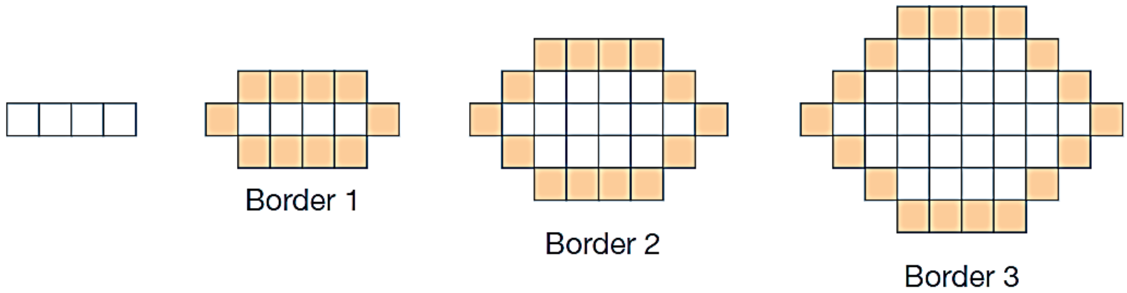
____

____

b. Pattern # 6 has a total of 625 blocks. How many white blocks are needed for pattern #6?

CRM Task 10

Fred has some colored kitchen floor tiles and wants to choose a pattern to make a border around white tiles. He generates patterns by starting with a row of four white tiles. He surrounds these four tiles with a border of colored tiles (Border 1). The design continues as shown below:



1. Fred writes the expression 4(b-1) + 10 for the number of tiles in each border, where b is the border number, b ≥1. Explain why Fred's expression is correct.

2. Emma wants to start with five tiles in a row. She reasons, "Fred started with four tiles and his expression was 4(b-1) + 10. So if I start with five tiles, the expression will be 5(b-1) + 10. Is Emma's statement correct? Explain your reasoning.

Conversions and Rates

Conversion - to change a value or expression from one form to another.
Rate - the comparison of two related quantities

Conversions: SMALL ⟶ LARGE

$$\frac{12 \text{ inches}}{1 \text{ foot}} \qquad \frac{3 \text{ feet}}{1 \text{ yard}} \qquad \frac{5280 \text{ feet}}{1 \text{ mile}}$$

$$\frac{1 \text{ foot}}{12 \text{ inches}} \qquad \frac{1 \text{ yard}}{3 \text{ feet}} \qquad \frac{1 \text{ mile}}{5280 \text{ feet}}$$

Ex.1 convert 65 inches to feet.

Ex.2 9432 feet to miles

Ex.3 Convert 1.3 inches

Ex.4 A student is reading a book at about 370 words per minute. Convert this rate to words per hour.

Ex.5 The average speed of a car on a stretch of interstate is 70 miles per hour. Convert this rate to feet per second.

Practice.

1. Convert 32 yards to inches.	2. Convert 4790 yards to miles.	3. Convert 2.3 hours to seconds.

4. An average typing speed for a person in a high school computer/typing class is about 44 words per minute. At this rate how many hours would it take to re-type a novel that is 45,000 words?	5. Craig Kimbrell of the Atlanta Braves, can throw his fastball at 102 miles per hour. Convert this rate to feet per second.

CRM Task 3 Felicias's Drive

As Felicia gets on the freeway to drive to her cousin's house, she notices that she is a little low on gas. There is a gas station at the exit she normally takes but she wonders if she will have to get gas before then. She normally sets her cruise control at the speed limit of 70mph and the freeway portion of the drive takes about an hour and 15 minutes. Her car gets about 30 miles per gallon on the freeway, and gas costs $3.50 per gallon.

a. Describe an estimate that Felicia might do in her head while driving to decide how many gallons of gas she needs to make it to the gas station at the other end.

b. Assuming she makes it, how much does Felicia spend per mile on the freeway?

CRM Task 4 Miles to Kilometers

The students in Mr. Sanchez's class are converting distances measured in miles to kilometers. To estimate the number of kilometers, Abby takes the number of miles, doubles it and then subtracts 20% of the result. Renato first divides the number of miles by 5 and then multiplies the result by 8.

a. Write an algebraic expression for each method.

b. Use your answer to part (a) to decide if the two methods give the same answer.

CRM Task 1 Bucky the Badger

http://blog.mrmeyer.com/?p=13514

Restate the Bucky the Badger problem in your own words:

Construct a viable argument for the following:
About how many total push-ups do you think Bucky did during the game?

Write down a number that you know is too high.

Write down a number that you know is too low.

What further information would you need to know in order to determine the exact number of total push-ups Bucky did in the course of the game?

If you're Bucky, would you rather your team score their field goals at the start of the game or the end?

What are some numbers of pushups that Bucky will never do in any game?