## Sets and Set Notation

- Set - a set is a collection of objects
- Subset - part of a larger set $\boldsymbol{A} \subset \boldsymbol{B}$
- Element - any object of a set
- Empty set - the set that has no elements
- Union - elements that are in both sets $A \cup B$
- Intersection - what sets share. $A \cap B$
- Complement - is the set of elements that are not in the set.

1. List the sample space for the spinners below using set notation.


$$
\begin{aligned}
M & =\{1,2,3,4,5,6,7\} \\
N & =\{1,3,5,7\}
\end{aligned}
$$

$$
\begin{gathered}
R=\{0,1,4,9,16,25\} \\
P=\{1,2,3,4,5\}
\end{gathered}
$$

2. What is $M \cap N$ ?
3. What is $M \cup N$ ?
4. Is $N \subset M$ ?
5. What is $R \cap P$ ?
6. What is $R \cup P$ ?
7. Is $P \subset R$ ?

## The Fundamental Counting Principal

If you have (a) ways of doing event 1 , (b) ways of doing event 2 , and (c) ways of doing event 3 , then you can find the total number of outcomes by multiplying.

$$
a \cdot b \cdot c
$$

8. Flipping a coin 3 times.
9. A jewelry store sells gold and platinum rings. Each ring is fitted with a ruby, sapphire, emerald, or diamond gemstone
10. Flipping a coin and rolling a dice.
11. How many different outfits can be made with 3 different jackets, 6 different shirts, and 4 different pairs of pants.
12. Three choices of sandwiches, three choices of sides, and 2 choices of drinks.
13. I license plate containing 2 digits followed by 5 letters.

## Introduction to Probability

- Probability is a number ( 0 to 1 ) that measures the chance that an event will occur.
- Outcome is the result of an experiment.
- Even - an outcome or collection of outcomes.
- Sample space - the set of all possible outcomes.
$P($ event $)=$ $\qquad$
Describing Probabilities
Can be written as a $\qquad$ , $\qquad$ or $\qquad$ .


1. Determine the probability for one roll of a fair number cube (1-6).

P (rolling a 2)

| $P$ (rolling an odd number) | $P$ (rolling at least a 5) |
| :--- | :--- |
|  |  |
|  |  |

$P$ (rolling a negative number)
2. Consider the letters in the state of NORTH CAROLINA. Suppose you took each letter of the word and put them into a bag. Find the probability of picking out the following at random.

| P (choosing an A) | P (choosing a vowel) | P (choosing an O or R) | P (choosing a letter) |
| :--- | :--- | :--- | :--- |
| 3. Use the spinner on the left to answer questions 12 - 20. <br> Write your answer as a fraction, decimal and a percent. | P (negative number) | P (odd number) |  |
| P (even number) |  |  |  |

Find the probability of the missing outcome.
4. There are three choices of soda - Coke, Sprite and Dr. Pepper. The probability of getting a Coke is $3 / 10$ and the probability of getting Sprite is $1 / 5$. Find the probability of getting a Dr. Pepper.
5. You sold the most for your school's fundraiser so you get to enter the cash vault. There are \$1, \$5, \$10 and $\$ 20$ bills. The probability of picking a $\$ 1$ bill is $48 \%$, a $\$ 5$ bill is $26 \%$, a $\$ 20$ bill is $10 \%$. What is the probability of picking a $\$ 10$ bill?

## Using Venn Diagrams

- A Venn diagram is an illustration of the relationships between and among sets, groups of objects that share something in common.

Shade in the appropriate area of the Venn Diagram.

1. $A \cap B$
2. $A \cap B^{\prime}$

3. $A \cap B \cap C$
4. $A \cap B^{\prime}$


A guidance counselor is planning schedules for 30 students. 16 want to take Spanish and 11 want to take Latin. 5 Say they want to take both. Display this information on the Venn Diagram below.
$\qquad$ 1. Find $P(S \cap L)$

$\qquad$ 2. Find $P(L)$
$\qquad$ 3. What is the probability that a student studies at least one subject? $P(S \cup L)$
$\qquad$ 4. What is the probability that a student studies exactly one subject?
$\qquad$ 5. What is the probability that a student studies neither subject? $P(S \cup L)$

- Compound Event - when two or more events occur
- Mutually Exclusive - no common outcomes
- Overlapping - common outcomes


## Mutually Exclusive

The probability that one or the other of several events will occur
is found by summing the individual probabilities of the events:

$$
P(A \text { or } B)=P(A)+P(B)
$$

Mutually Exclusive Events

$P(A$ or $B)=P(A)+P(B)$

1. Find the probability that a girl's favorite department store is Macy's or Nordstrom.
2. Find the probability that a girl's favorite store is not JC Penny's.

| Macy's | 0.25 |
| :--- | :--- |
| Saks Fifth Ave. | 0.20 |
| Nordstrom | 0.20 |
| JC Penny's | 0.10 |
| Bloomingdale's | 0.25 |

3. When rolling two dice, what is probability you roll a double?
4. When rolling two dice, what is probability that your sum will be 4 or 5 ?
5. When rolling two dice, what is probability that your

| + | 1 | 2 | 3 | 4 | 5 | 6 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| 1 |  |  |  |  |  |  |
| 2 |  |  |  |  |  |  |
| 3 |  |  |  |  |  |  |
| 4 |  |  |  |  |  |  |
| 5 |  |  |  |  |  |  |
| 6 |  |  |  |  |  |  | sum will be even or a sum of 9 ?

6. What is the probability of picking a spade?
7. What is the probability of picking a queen or an ace from a deck of cards?

| Clubs | Spades | Hearts | Diamonds |
| :---: | :---: | :---: | :---: |
| A* | A | ${ }^{\text {a }}$ | A* |
| 2* | $2{ }^{*}$ | 2 | 2 |
| 3. | 3. | $3 \vee$ | 3 |
| 4* | 44 | 4 | 4. |
| 5* | 54 | 5 | 5 |
| 64 | 64 | 6 | 64 |
| 7* | 74 | 7 | 7 |
| $8 *$ | 84 | 8 | $8{ }^{8}$ |
| 9* | 94 | 9 | 9 |
| 10* | 104 | $10 \vee$ | 104 |
| Jack* | JackA | Jack ${ }^{\text {\% }}$ | Jack* |
| Queen* | Queen ${ }^{\text {a }}$ | Queen V | Queen* |
| King* | KingA | King ${ }^{\text {V }}$ | King* |

## Overlapping Events

Probability that non-mutually exclusive events $A$ and $B$ or both will occur expressed as:

$$
P(A \text { or } B)
$$

$$
P(A \cup B)=P(A)+P(B)-P(A \cap B)
$$

Non-Mutually Exclusive Events


$$
\mathrm{P}(\mathrm{~A} \text { or } \mathrm{B})=\mathrm{P}(\mathrm{~A})+\mathrm{P}(\mathrm{~B})-\mathrm{P}(\mathrm{~A} \text { and } \mathrm{B})
$$

1. Find the probability that a person own a cat or dog.

2. Find the probability that a person drinks tea and coffee.

3. Find the probability that person watches the US Open, Wimbledon, and Australian Open.

4. Find the probability of picking a female or a person from Florida out of the committee members.

|  | Female | Male |
| :---: | :---: | :---: |
| Florida | 8 | 4 |
| Alabama | 6 | 3 |
| Georgia | 7 | 3 |

5. Find the probability of picking a king or a club in a deck of cards.

Standard Deck of 52 Playing Cards

| Clubs | Spades | Hearts | Diamonds |
| :---: | :---: | :---: | :---: |
| A* | A ${ }^{\text {a }}$ | ${ }^{\text {a }}$ | ${ }^{4}$ |
| 2* | 2. | $2 v$ | 2 |
| 3* | 34 | 3 V | 34 |
| 4. | 44 | 4 | $4 *$ |
| 54 | 54 | 5 | 5 |
| 64 | 64 | ${ }^{6}$ | 6 |
| 7* | 74 | 7 | 7 |
| $8 *$ | 84 | 8 | 8 |
| 94 | 94 | 9 | 9 |
| 10* | $10 \wedge$ | $10 \vee$ | 104 |
| Jack* | JackA | Jack | Jack* |
| Queent | Queen ${ }^{\text {a }}$ | Queen ${ }^{\text {V }}$ | Queen* |
| King* | KingA | King ${ }^{\text {b }}$ | King* |

6. When rolling 2 dice, what is the probability of getting an even sum or a number greater than 10
7. When rolling 2 dice, what is the probability of getting double or a sum of 6 ?

| + | 1 | 2 | 3 | 4 | 5 | 6 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 1 |  |  |  |  |  |  |
| 2 |  |  |  |  |  |  |
| 3 |  |  |  |  |  |  |
| 4 |  |  |  |  |  |  |
| 5 |  |  |  |  |  |  |
| 6 |  |  |  |  |  |  |

Determine if the scenario is mutually exclusive or overlapping.

1. A spinner has an equal chance of landing on each of its eight numbered regions. After spinning, it lands in region three or six.
2. A bag contains six yellow jerseys numbered one to six. The bag also contains four purple jerseys numbered one to four. You randomly pick a jersey. It is purple or has a number greater than five
3. A magazine contains twelve pages. You open to a random page. The page number is eight or ten.
4. A box of chocolates contains six milk chocolates and four dark chocolates. Two of the milk chocolates and three of the dark chocolates have peanuts inside. You randomly select and eat a chocolate. It is a milk chocolate or has no peanuts inside.

## Conditional Probability

- Contains a condition that limits the sample space of an event.
- Written as $\mathrm{P}(\mathrm{A} \mid \mathrm{B})$ the probability of event A , given event B .
- $P(A \mid B)=\frac{P(A \cap B)}{P(B)}$

| EYE COLOR | Black | Brown | Blue | Green | Gray | Total |
| :--- | ---: | ---: | ---: | ---: | ---: | ---: |
| Female | 20 | 30 | 10 | 15 | 10 | 85 |
| Male | 25 | 15 | 12 | 20 | 10 | 82 |
| Total | 45 | 45 | 22 | 35 | 20 | 167 |

1. P(Blue eyes)
2. P(Female)
3. P(Green and Male)
4. P(Brown and Female)
5. P(Gray or Female)
6. P(Male or Black)
7. (Male | Blue)
8. (Gray | Female)

## Independent and Dependent Events

## Independent Events

- Event A occurring does NOT affect the probability of Event B occurring.
- $P(A$ and $B)=P(A \cap B)=P(A) \bullet P(B)$

| $\mathrm{P}(\mathrm{A})=0.8 \quad \mathrm{P}(\mathrm{B})=0.25 \quad \mathrm{P}(\mathrm{C})=0.6$ |
| :--- |
| 1. Find $\mathrm{P}(\mathrm{A}$ and C$)$ |
|  |

## Dependent Events

- Event A occurring AFFECTS the probability of Event B occurring.
- Usually you will see the words "WITHOUT REPLACING."
- $P(A$ and $B)=P(A \cap B)=P(A) \bullet P(B \mid A)$

7. A jar contains 3 red, 5 green, 2 blue and 6 yellow marbles. A marble is chosen at random from the jar. A second marble is chosen without replacing the first one. What is the probability of choosing a green and a yellow marble?
8. An aquarium contains 6 male goldfish and 4 female goldfish. You randomly select a fish from the tank, do not replace it, and then randomly select a second fish. What is the probability that both fish are male?
9. James has 3 dimes, 4 pennies, and 2 quarters in his pocket. If each coin is equally likely to be pulled out of his pocket in order without replacement quarters in a row first?
10. In a cookie jar there are 10 chocolate chip cookies and 8 peanut butter cookies left. The cookies are randomly mixed in the jar. What is the probability of pulling two of the same types of cookies out of the cookie jar in a row

## How to Determine If 2 Events Are Independent:

- Substitute in what you know in to $\mathbf{P}(\mathbf{A} \cap \mathbf{B})=\mathbf{P}(\mathbf{A}) \bullet \mathbf{P}(\mathbf{B})$ and check to see if left side equals right side.
- If it's equal, then it's independent.
- If it's not equal, then it's not independent (or dependent).

$$
\begin{aligned}
& \qquad P(M)=0.8 \quad P(N)=0.25 \quad P(R)=0.6 \\
& \hline \text { 11. If the probability of } P(M \text { and } N)=0.2, \text { are } \mathrm{M} \text { and } \\
& \mathrm{N} \text { independent? }
\end{aligned} \begin{aligned}
& \text { 12. If the probability of } P(N \text { and } R)=0.3, \text { are } \mathrm{N} \text { and } \mathrm{R} \\
& \text { independent? }
\end{aligned}
$$

13. Let event $\mathrm{M}=$ taking a math class. Let event $\mathrm{S}=$ taking a science class. Then, M and $\mathrm{S}=$ taking a math class and a science class. Suppose $P(M)=0.6, P(S)=0.5$, and $P(M$ and $S)=0.3$. Are M and S independent?
14. In a class, $60 \%$ of the students are female. $50 \%$ of all students in the class have long hair. $45 \%$ of the students are female and have long hair. Of the female students, $75 \%$ have long hair. Let $F$ be the event that the student is female. Let L be the event that the student has long hair. One student is picked randomly. Are the events of being female and having long hair independent?
