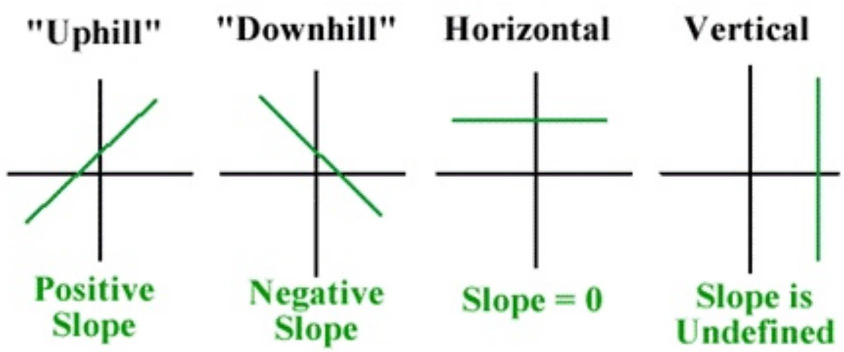
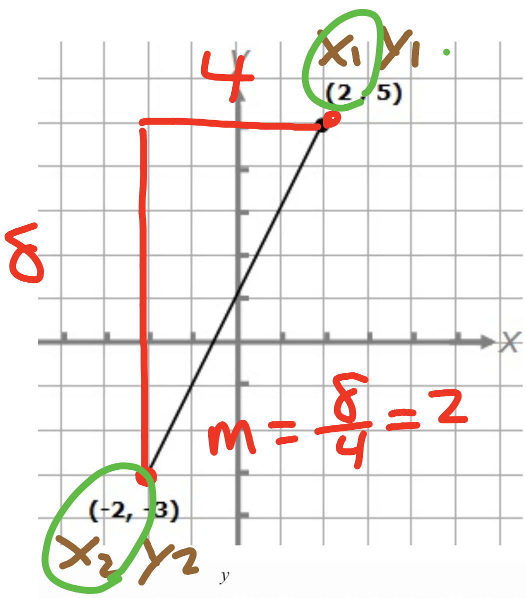
Slope - is the steepness of a line

$$\text{Slope} = \frac{\text{rise}}{\text{run}}$$

$$\text{Slope Formula} = \frac{y_2 - y_1}{x_2 - x_1}$$



Ex.1

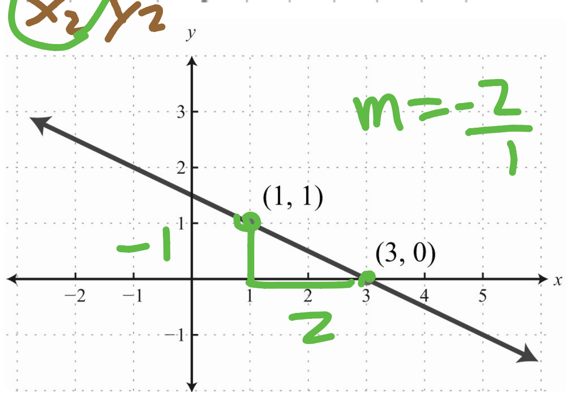


$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

$$\frac{-3-5}{-2-2}$$

$$\frac{-8}{-4} = \boxed{2}$$

Ex.2



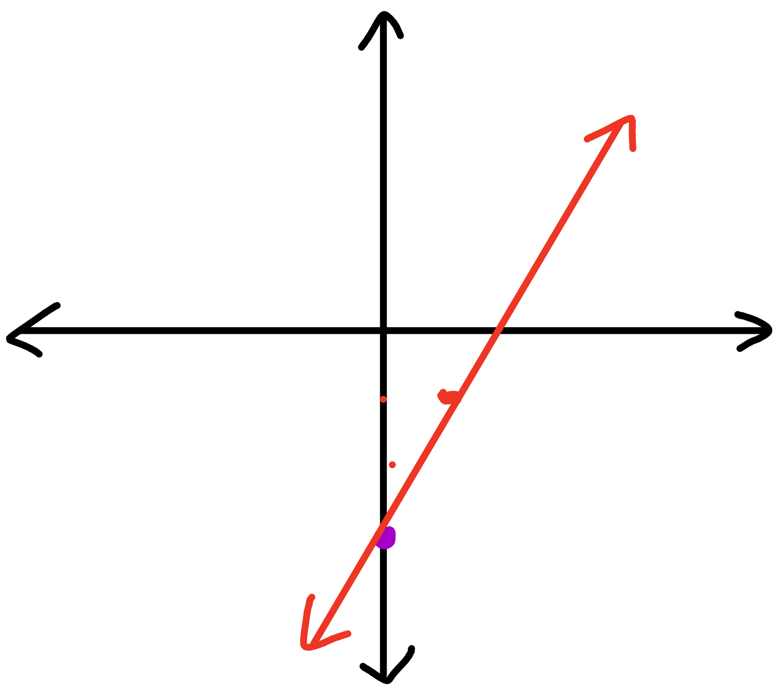
Slope intercept Form

$$y = mx + b$$

slope y-intercept

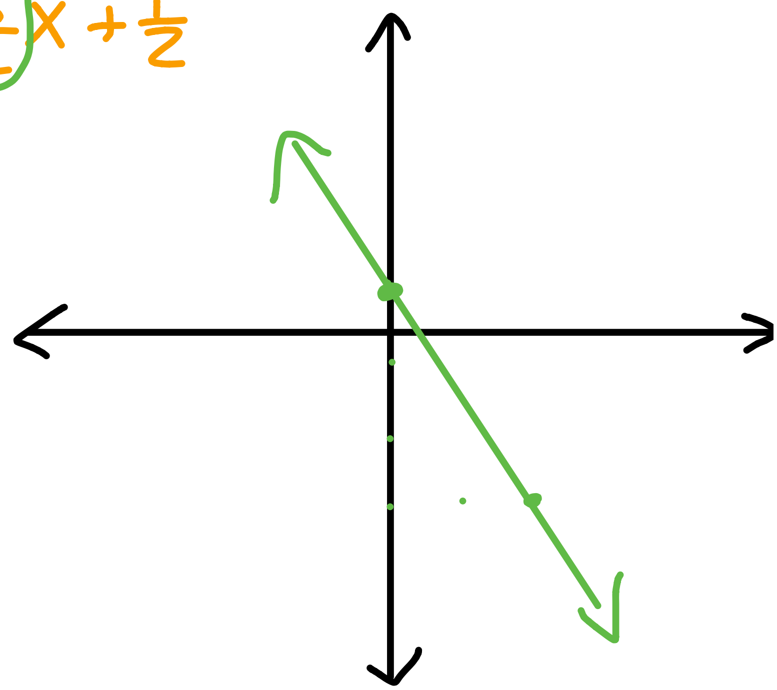
$$y = 3x - 5$$

Ex.1 graph $y = \frac{2}{1}x - 3$



Ex.2 graph $\frac{\cancel{2}y}{\cancel{2}} = \frac{-3x}{2} + \frac{1}{2}$

$y = -\frac{3}{2}x + \frac{1}{2}$



Ex.3 graph $y = 3$ $x = -1$

Finding parallel lines

1. Put the equation in slope intercept form.

   $y = mx + b$

2. Take out the slope (m) and plug in your point (x,y) into the point slope formula.

   $y - y_1 = m(x - x_1)$

Ex.1 write the equation of the line that passes through (-1,6) and is parallel to the graph

$x_1 \; y_1$

$y = 2x + 4$

$m = 2$

$y - y_1 = m(x - x_1)$

$y - 6 = 2(x + 1)$

$y - \cancel{6} = 2x + 2$

$\quad \cancel{+6} \qquad\quad +6$

$\boxed{y = 2x + 8}$

Finding perpendicular lines

1. Put the equation in slope intercept form.

$y=mx+b$

2. Take the opposite reciprocal of the slope.

$m=-\frac{2}{3}$ $m=-\frac{3}{2}$ $m=\frac{-5}{1}$ $m=\frac{1}{5}$

3. Take out the slope (m) and plug in your point (x,y) into the point slope formula.

$y-y_1=m(x-x_1)$

Ex.2 write the equation of the line that passes through (-2,8) and is perpendicular to

$x_1\ y_1$

$y=4x+1$

$m=-\frac{1}{4}$

$y-y_1=m(x-x_1)$

$y-8=-\frac{1}{4}(x+\frac{2}{1})$

$y-\cancel{8}=-\frac{1}{4}x-\frac{1}{2}$

$+\cancel{8}$ $+8$

$\boxed{y=-\frac{1}{4}x+\frac{15}{2}}$ ← 7.5

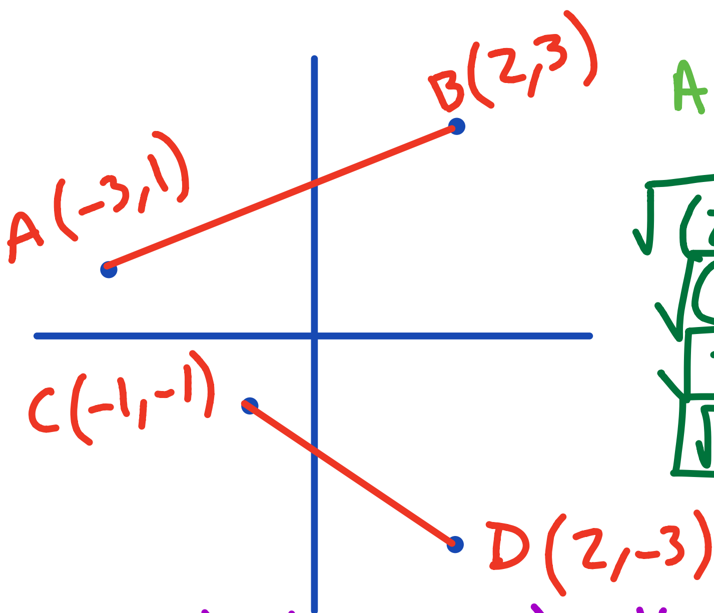
$\frac{16}{2}-\frac{1}{2}=7.5$

The distance formula

The distance between two points is...

$$d=\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$$

Ex.1 Find the length of $\overline{AB}$ and $\overline{CD}$.



x_1 y_1 $\quad$ x_2 y_2

A(-3,1) $\quad$ B(2,3)

$$\sqrt{(2+3)^2+(3-1)^2}$$
$$\sqrt{(5)^2+(2)^2}$$
$$\sqrt{25+4}$$
$$\boxed{\sqrt{29}\approx 5.4}$$

x_1 y_1 $\quad$ x_2 y_2

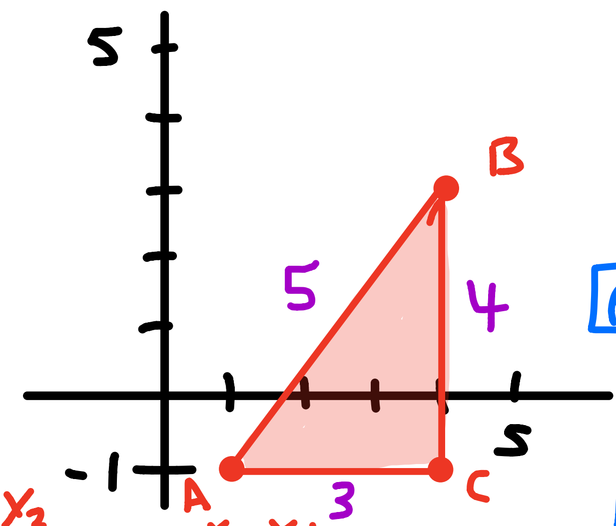
D(2,-3) $\quad$ C(-1,-1)

$$d=\sqrt{(-1-2)^2+(-1+3)^2}$$
$$\sqrt{(-3)^2+(2)^2}$$
$$\sqrt{9+4}$$
$$\boxed{\sqrt{13}\approx 3.6}$$

Calculating Perimeter and Area on the coordinate plane

- Calculate the distance of each side
- Then use the appropriate formula

Ex.1 Find the perimeter and area of the triangle. A(1,-1), B(4,3), and C(4,-1).



$x_2\ y_2$ $\quad$ $x_1\ y_1$

A(1,-1) $\quad$ B(4,3)

$$d = \sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$$

$$\sqrt{(1-4)^2+(-1-3)^2}$$

$P = 5+4+3$

$\boxed{P = 12}$

$A = \frac{1}{2}b \cdot h$

$= \frac{1}{2}(3)(4)$

$\boxed{A = 6}$

$$\sqrt{(-3)^2 + (-4)^2}$$

$$\sqrt{9+16}$$

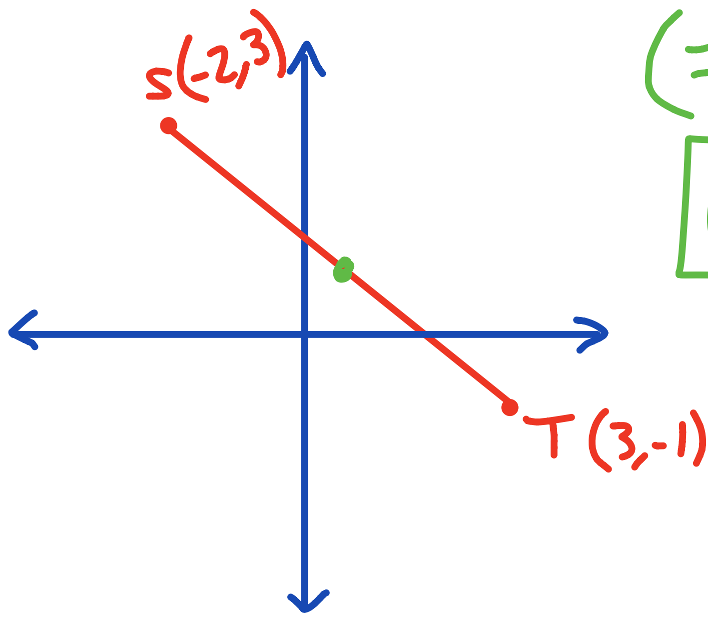
$$\sqrt{25} = \boxed{5}$$

Finding midpoint

The midpoint M of $\overline{AB}$ with endpoints A and B is found by...

$$M\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$$

Ex.1 Find the midpoint of $\overline{ST}$.



$$\left(\frac{-2+3}{2}, \frac{3-1}{2}\right)$$

$$\boxed{\left(\frac{1}{2}, 1\right)}$$

Ex.2 Find the endpoint of $\overline{PR}$ that has the midpoint P(-4,1) and the endpoint Q(2,-3).

-6 (2,-3) $+4$

(-4,1)

-6 $\boxed{(-10, 5)}$ $+4$

<u>Partitioning Segments</u>

1. Calculate the difference between x-values then the y-values.
2. Multiply the difference by the ratio
3. Add the product to the first x or y value.

Ex.1 calculate the midpoint of: (-2,1) and (4,10).

+3 +4.5

(-2,1)

$\frac{1}{2}(+6)$ $\boxed{(1, 5.5)}$ $(+9)\frac{1}{2}$

(4,10)

Ex.2 show that (0,8) is the midpoint of the line segment with the end points (-2,11) and (2,5).

(-2,11)

+2 (0,8) -3

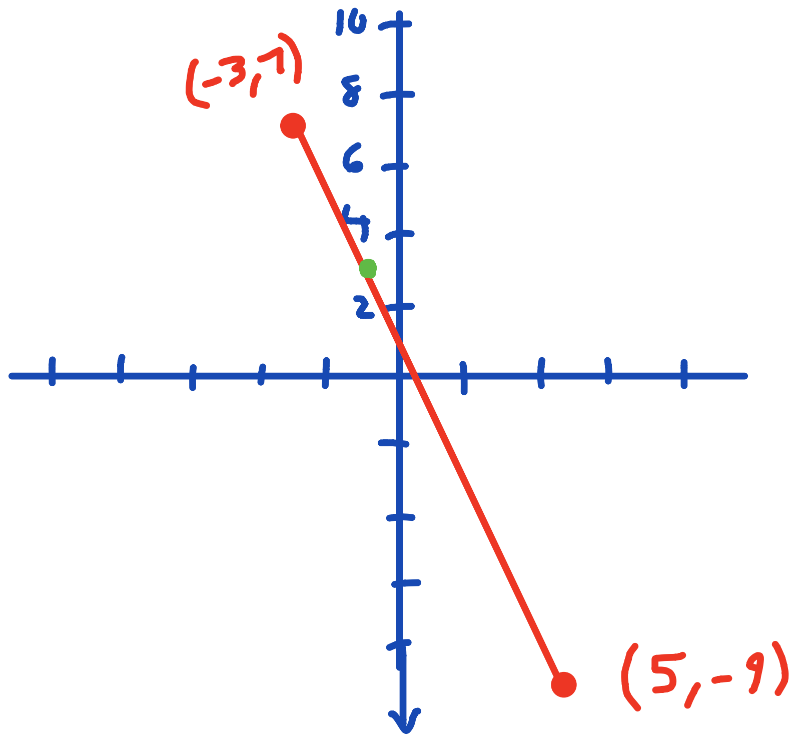
+2 (2,5) -3

Ex.3 Determine the point that is 1:3 the distance from the endpoint (-3,7) of the segment with the end points (-3,7) and (5,-9).

$$\frac{1}{4}(+8) \qquad (\overset{+2}{-3},\ \overset{-4}{7}) \qquad (-16)\frac{1}{4} \quad \frac{1}{4}$$

$$\boxed{(-1,3)}$$

$$(5,-9)$$



Equations of Circles

Standard form: $(x-h)^2+(y-k)^2=r^2$

center: (h,k) radius $= r$

General form:

$$ax^2+by^2+cx+dy+e=0$$

Ex.1 Write the equation of a circle with center (3,-2) and a radius of 4. r

h k

$$(x-h)^2+(y-k)^2=r^2$$

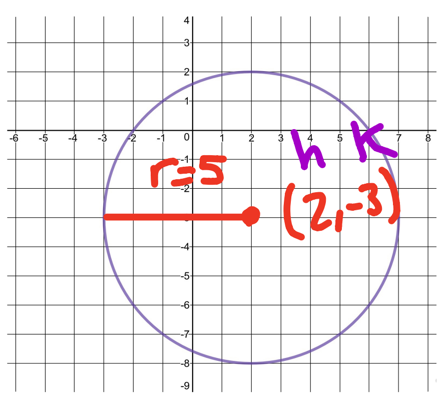
$$(x-3)^2+(y+2)^2=16$$

Ex.2 Find the coordinates of the center and measure of the radius.

$$(x-6)^2+(y+3)^2=\sqrt{25}$$

center $(6,-3)$ $r=5$

Ex.3 Find the equation of the circle.



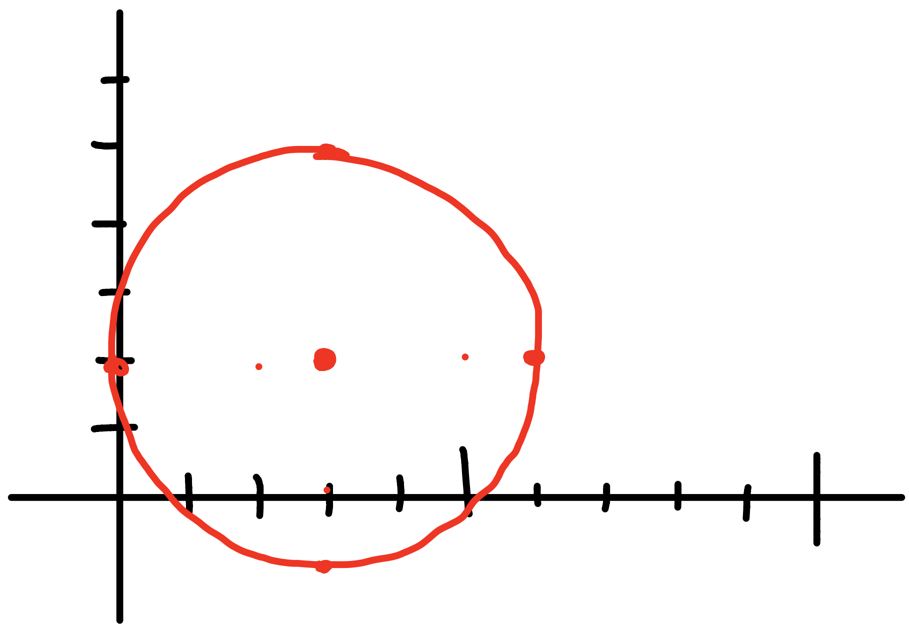
$$(x-h)^2+(y-k)^2=r^2$$

$$(x-2)^2+(y+3)^2=25$$

Ex.4 Graph the circle.

$(x-3)^2+(y-2)^2=9$

Center $(3,2)$ $r=3$



Converting from general to standard form.

1. (a) needs to be one. ✓
2. Move the x and y terms together ✓
3. Move (e) to the other side.
4. Complete the square.
5. factor the left side and simplify.

$$ax^2+by^2+cx+dy+e=0$$

Ex.1 $x^2+y^2-8x+7=0$

$$x^2-8x+y^2+\cancel{7}=0$$
$$\cancel{-7} \qquad -7$$

$$x^2-8x+(-4)^2+y^2=-7+(-4)^2$$

$$\boxed{x^2-8x+16}+y^2=9$$

a·c
1·16
16
1·16
2·8
-4·-4

$$(x-4)(x-4)+y^2=9$$

$$(x-4)^2+y^2=9$$

center $(4,0)$ $r=3$

Ex.2 $x^2+y^2+4x-6y-3=0$

$x^2+4x+y^2-6y=3$

$(x+2)^2+(y-3)^2=3+2^2+3^2$

$\boxed{(x+2)^2+(y-3)^2=16}$

Ex.3 $\frac{\cancel{2}x^2}{\cancel{2}}+\frac{\cancel{2}y^2}{\cancel{2}}-\frac{16x}{2}+\frac{4y}{2}+\frac{20}{2}=\frac{0}{2}$

$x^2+y^2-8x+2y+10=0$

$x^2-8x+y^2+2y=-10$

$(x-4)^2+(y+1)^2=-10+4^2+1^2$

$(x-4)^2+(y+1)^2=7$

center $(4,-1)$ $r=\sqrt{7}$

Ex.4 standard to general form.

$(x-4)^2+(y+3)^2=36$

$(x-4)(x-4)+(y+3)(y+3)=36$

$x^2-4x-4x+16+y^2+3y+3y+9=\cancel{36}$

$-36 \quad -\cancel{36}$

$\boxed{x^2+y^2-8x+6y-11=0}$

Coordinate Geometry Proofs

<u>Parallelograms</u>

(1) Parallelograms opposite sides are congruent.

(2) opposite angles are congruent, the consecutive angles are supplementary.

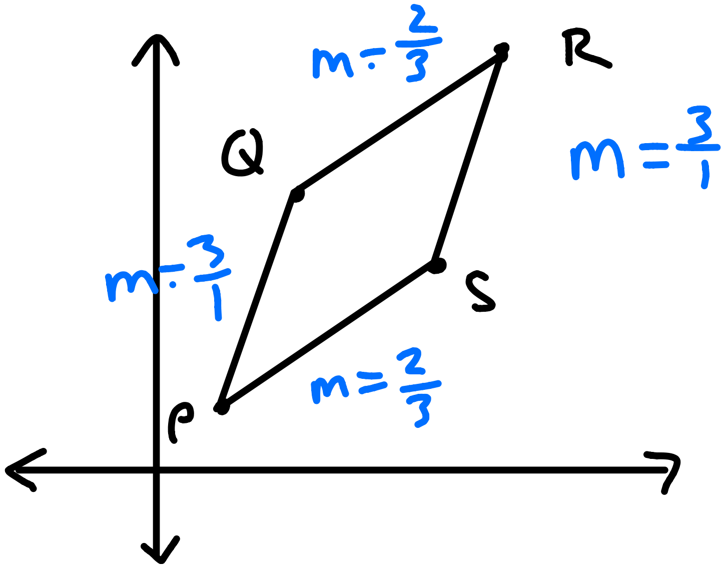
(3) the diagonals bisect each other.

(4) the diagonal of a parallelogram forms two congruent triangles.

<u>How to prove that a quadrilateral is a parallelogram.</u>

- If the slopes of opposite sides are the same, then the opposite sides are parallel, therefore the quadrilateral is a parallelogram.
- If the distance of the opposite sides are equal, than the quadrilateral is a parallelogram.

Ex.1 Prove that the quadrilateral with the coordinates P(1,1), Q(2,4), R(5,6), and S(4,3).

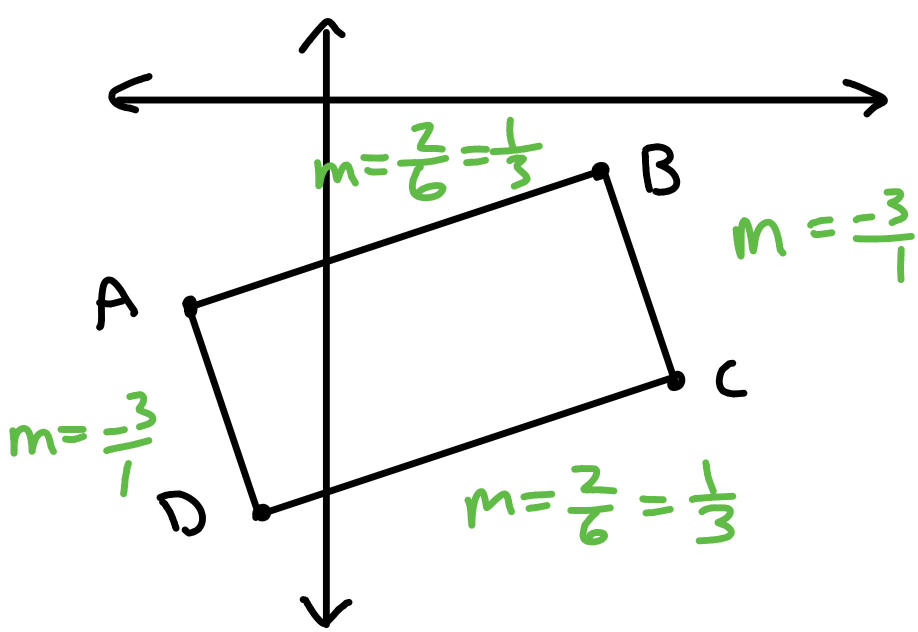


The opposite slopes are the same, therefore the quadrilateral is a parallelogram.

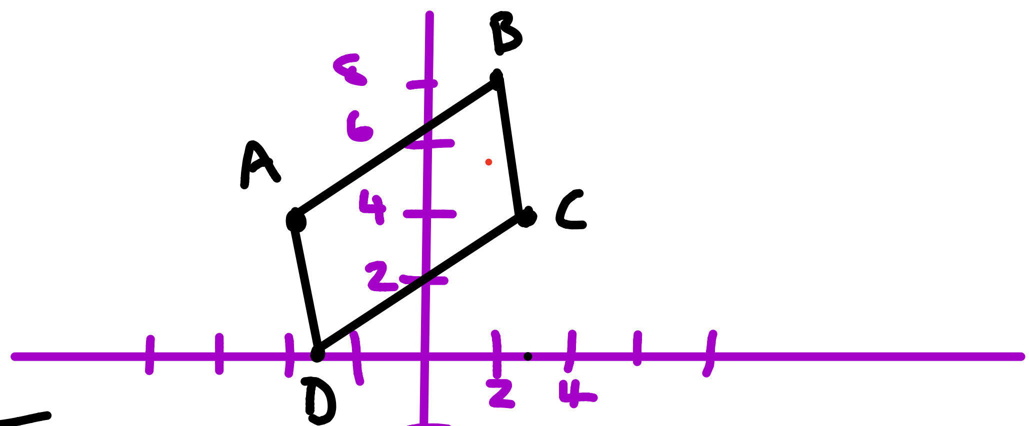
How to prove that a parallelogram is a rectangle.

- By slopes - if two consecutive sides are opposite reciprocals, than the sides are perpendicular. Therefore the parallelogram is a rectangle.
- By diagonals - if the diagonals have the same distance, than the parallelogram is a rectangle.

Ex.2 Prove that the quadrilateral with the coordinates A(-1,-1), B(5,-1), C(5,-4), D(-1,-4).



Ex.3 Using slope and distance, Prove if the quadrilateral is a parallelogram. A(-4,4), B(2,8), C(3,4), D(-3,0).



$\overline{AB}=\sqrt{52}$ $\quad m=\frac{2}{3}$

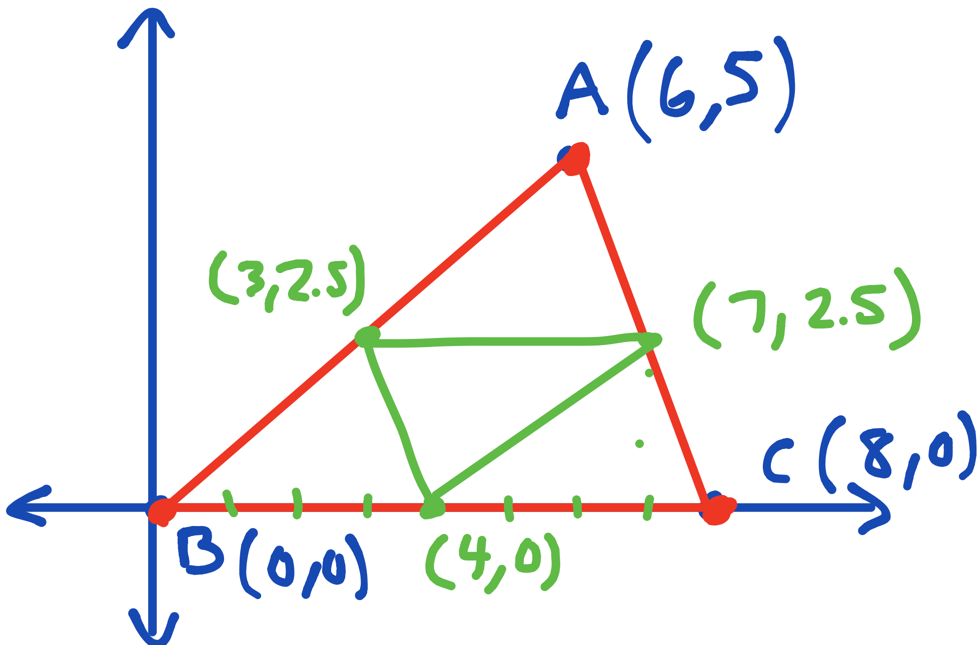
$\overline{BC}=\sqrt{17}$ $\quad m=-4$

$\overline{CD}=\sqrt{52}$ $\quad m=\frac{2}{3}$

$\overline{AD}=\sqrt{17}$ $\quad m=-4$

Ex.4 Find the midpoints of the triangle.

$$\text{midpoint} = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$$



$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

Finding the shortest distance from a given point to a line.

1. Put the equation in:

$$ax + by + c = 0$$

2. Then plug in your point (x,y) and a, b, and c into:

$$d = \frac{|ax + by + c|}{\sqrt{a^2 + b^2}}$$

Ex.1 $\underset{x}{(2},\underset{y}{1)}$ to $-2x + y = \cancel{1}$

$-1 \quad \cancel{-1}$

$$-2x + y - 1 = 0$$

$$ax + by + c = 0$$

$$a = -2 \quad b = 1 \quad c = -1$$

$$d = \frac{|ax + by + c|}{\sqrt{a^2 + b^2}} = \frac{|(-2)(2) + (1)(1) + (-1)|}{\sqrt{(-2)^2 + (1)^2}}$$

$$= \frac{|-4 \cancel{+1} \cancel{-1}|}{\sqrt{4 + 1}}$$

$$= \frac{4}{\sqrt{5}}$$

$$\boxed{d = 1.79}$$