Introduction to Transformations

- Point- an exact position or location in a given plane.

  • P

- Line- the set of points A and B in a plane and the infinite number of points that continue beyond the points.

  A   B $\overleftrightarrow{AB}$

- Line segment- a line with two end points.

  A   B $\overline{AB}$

- Ray- is defined by two points, one is an endpoint.

  A   B $\overrightarrow{AB}$

- Angle- is formed where two line segments or rays share an end point.

  A
  B
  C $\angle ABC$

Types of Angles

Acute angle- measures less than 90 degrees but greater than zero degrees.

Obtuse angle- measures greater than 90 degrees but less than 180 degrees.
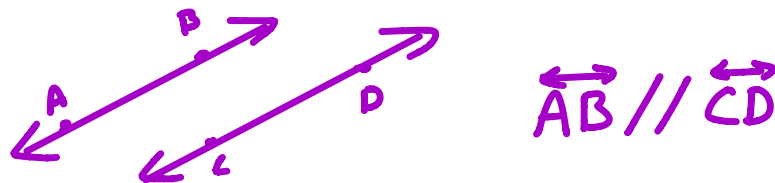
Right angle- measure exactly 90 degrees.

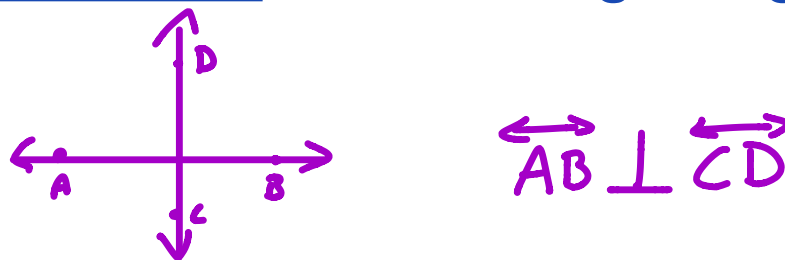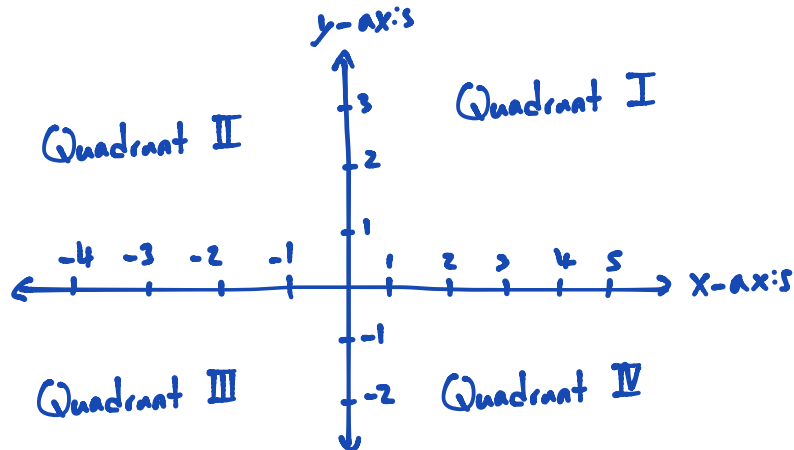**Straight**- an angle that measure 180 degrees

Types of Lines

Parallel lines- are two lines that will never intersect.

$$\overleftrightarrow{AB} \, // \, \overleftrightarrow{CD}$$

Perpendicular lines- meet at a right angle.

$$\overleftrightarrow{AB} \perp \overleftrightarrow{CD}$$

Line of symmetry- is a line separating a figure into two halves that are mirror images.

Types of transformations

Transformation- changes the position, shape, or size of a figure on a coordinate plane.

Preimage- the original figure
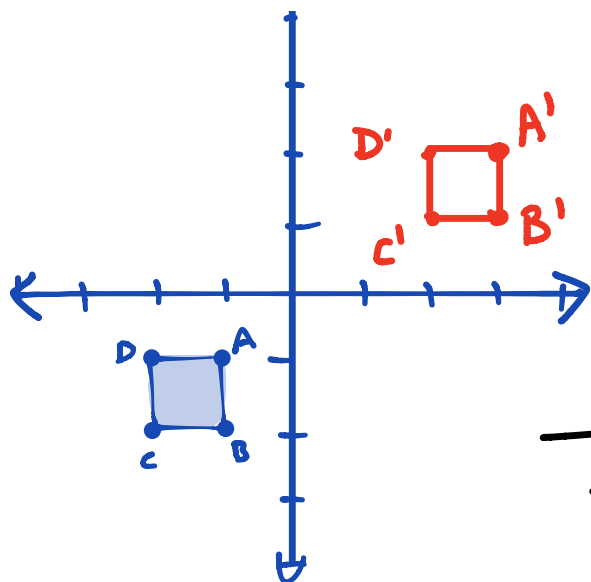
Image- resulting figure.

Isometric- transformations that are congruent; rotations, reflections, and translations.

Congruent- same size, shape, and angle.

Translation- a transformation that moves each point of a figure the same distance in the same direction.

Ex.1 Translations - right 4 and up 3

$$T_{4,3}(X,Y)$$

$A(-1,-1) \rightarrow A'(3,2)$
$B(-1,-2) \rightarrow B'(3,1)$
$C(-2,-2) \rightarrow C'(2,1)$
$D(-2,-1) \rightarrow D'(2,2)$

$$T_{h,k}(X,Y) \Rightarrow (X+h, Y+K)$$

Ex.2 Translate the image by (x-2, y+7) $T_{-2,7}$

$A(0,7) \rightarrow A'(-2,14)$

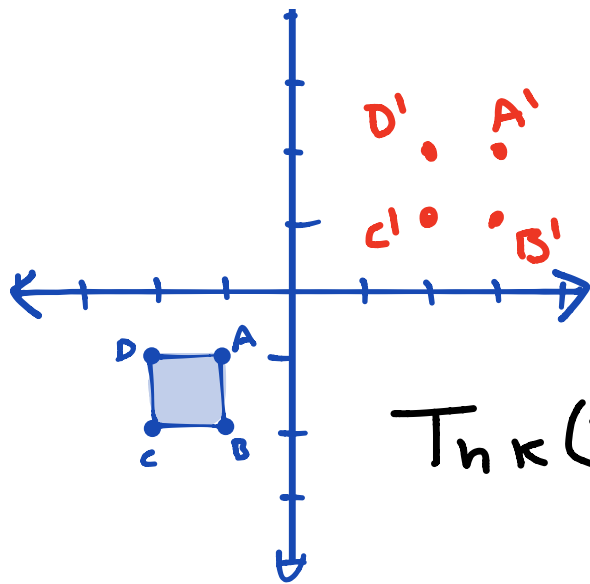$B(-2,-3) \rightarrow B'(-4,4)$

$C(5,0) \Rightarrow C'(3,7)$

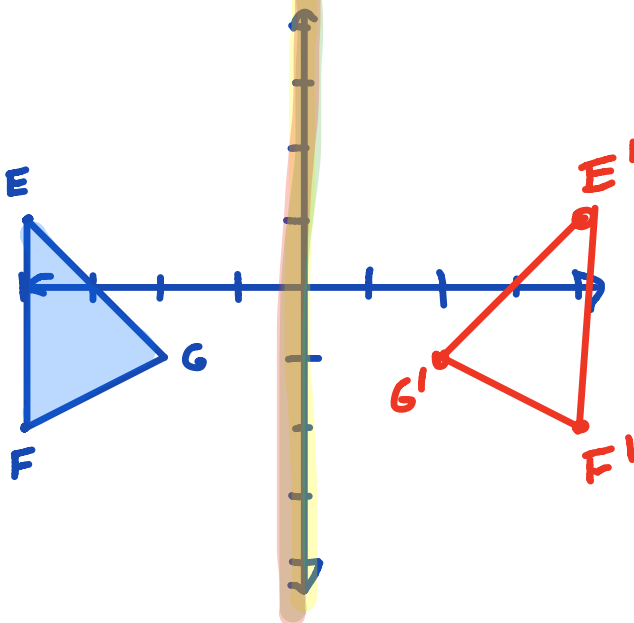**Reflection across the x-axis**

•B

$A(1,1) \Rightarrow A'(1-1)$

$$A(-1,-1) \Rightarrow A'(3,2)$$
$$B(-1,-2) \Rightarrow B'(3,1)$$
$$C(-2,-2) \Rightarrow C'(2,1)$$
$$D(-2,-1) \Rightarrow D'(2,2)$$

$$T_{hk}(x,y) \Rightarrow (x+h, y+k)$$

Reflection- a transformation where a mirror image is created

Reflection across the y-axis

$$E(-4,1) \Rightarrow E'(4,1)$$
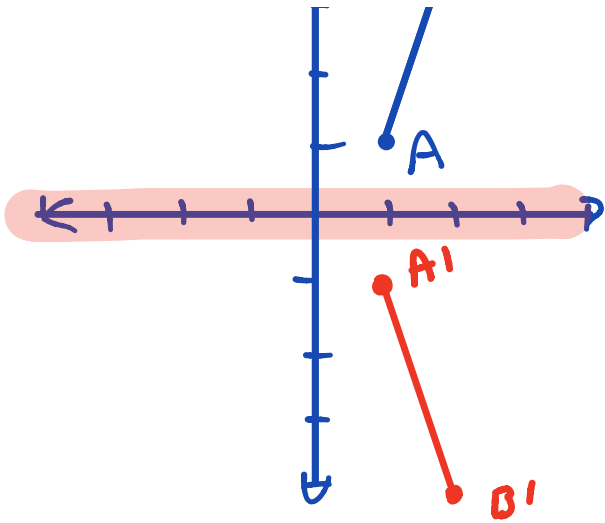$$F(-4,-2) \Rightarrow F'(4,-2)$$
$$G(-2,-1) \Rightarrow G'(2,-1)$$

$$r_y(x,y) \rightarrow (-x,y)$$

$$r_x(x,y) \rightarrow (x,-y)$$

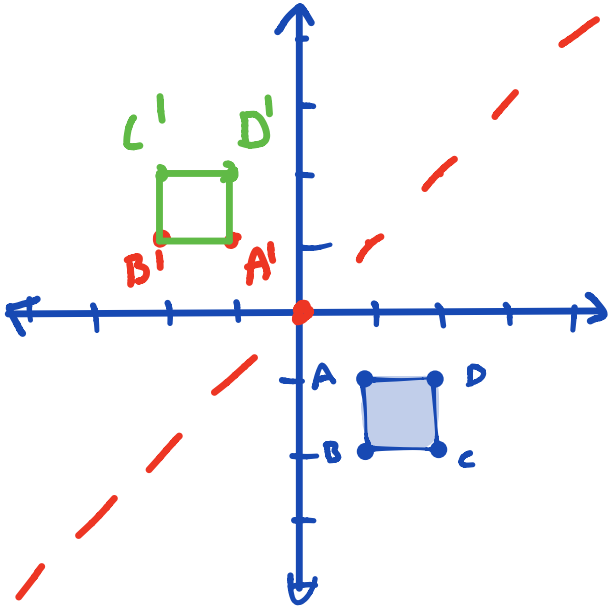## Reflection across the x-axis

$$A(1,1) \Rightarrow A'(1,-1)$$

$B(2,4) \Rightarrow B'(2,-4)$

$r_x(\underset{\bigcirc}{X}, y) \Rightarrow (X, -y)$

Reflection across y = x

$y = mx + b$
$y = x$

$A(1,-1) \Rightarrow A'(-1,1)$
$B(1,-2) \Rightarrow B'(-2,1)$
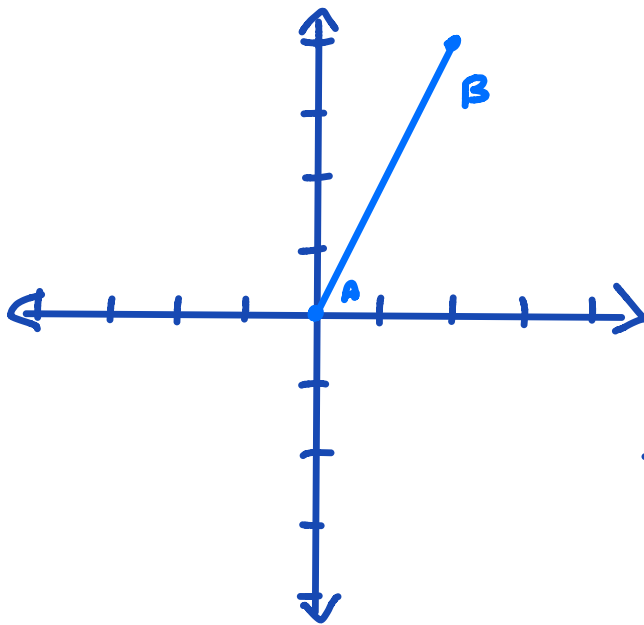$C(2,-2) \Rightarrow C'(-2,2)$
$D(2,-1) \Rightarrow D'(-1,2)$

$r_{y=x}(X, y) \Rightarrow (y, x)$

$r_{y=-x}(X, y) \Rightarrow (-y, -x)$

Rotation- a transformation that turns a figure around a fixed center point. Origin (0,0)

Clockwise- rotating a figure in the direction that the hands on the clock move

Counterclockwise- rotating a figure in the opposite direction that the hands on a clock move.

90° CCW

$A(0,0) \rightarrow A'(0,0)$

$B(2,4) \rightarrow B'(-4,2)$

$R_{90}(X,Y) \Rightarrow (-Y, X)$

180° CCW

$A(0,0) \rightarrow A'(0,0)$

$B(2,4) \rightarrow B'(-2,-4)$

$R_{180}(X,Y) \Rightarrow (-X,-Y)$

270° CCW

$A(0,0) \rightarrow A'(0,0)$

$B(2,4) \rightarrow B'(4,-2)$

$R_{270}(X,Y) \Rightarrow (Y,-X)$

Rotations CCW

$R_{90}(X,Y) \Rightarrow (-Y, X)$

$R_{180}(X,Y) \Rightarrow (-X,-Y)$

$R_{270}(X,Y) \Rightarrow (Y,-X)$

Dilations

- **In a dilation, we are enlarging and reducing the pre-image**
- **Dilations are not isometric.**
- **When we dilate an image, the size changes, the angles do not.**
- **This is the one transformation where the pre-image and image are similar, but not congruent.**

Scale factor:

- **We use "k" to represent scale factor.**
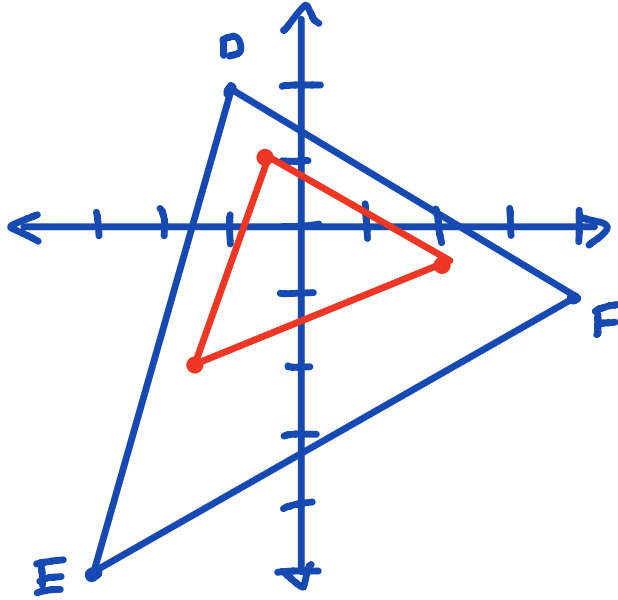- **We multiply by k to find the image.**

$$K = \frac{image}{preimage}$$

Ex.1 Dilate the image by k=2 enlargement
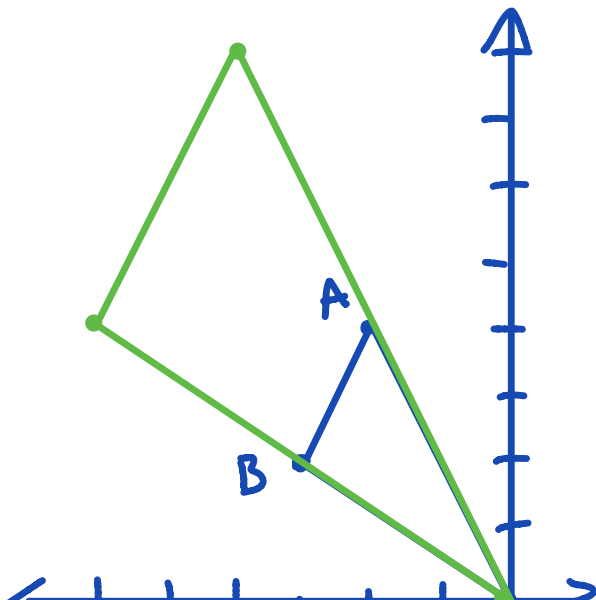
$A(-2,4) \rightarrow A'(-4,8)$

$B(-3,2) \rightarrow B'(-6,4)$

$C(0,0) \rightarrow C'(0,0)$

Ex.2 Dilate the image by k=1/2. reduction

$D(-1,2) \rightarrow D'(-.5,1)$
$E(-3,-5) \rightarrow E'(-1.5,-2.5)$
$F(4,-1) \rightarrow F'(2,-.5)$