Proving Theorems about Lines and Angles

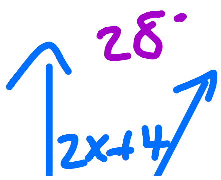
Angle Vocabulary

- Complementary- two angles whose sum is 90 degrees.
- Supplementary- two angles whose sum is 180 degrees.
- Congruent angles- two or more angles with the same measure.
- Angle bisector- a ray or a line segment that divides an angle into two congruent angles.
- Vertical angles- are nonadjacent angles formed by two pairs of opposite rays. vertical angles are congruent.
- Linear pair- two adjacent angles whose non shared sides form a straight angle. Linear pairs are supplementary.

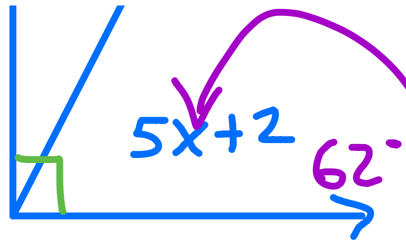
Ex.1 angle one and two are complementary. Solve for x and the measure of both angles.

$\angle 1 = 5x+2$

$\angle 2 = 2x+4$



$2x+4+5x+2=90$



$7x + \cancel{6} = 90$

$\quad -6 \quad -6$

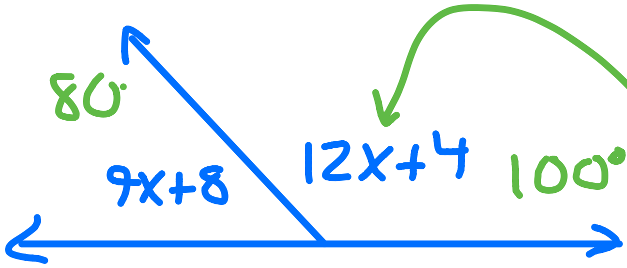
$\frac{\cancel{7}x}{\cancel{7}} = \frac{84}{7}$

$\boxed{x = 12}$

Ex.2 angle one and two are supplementary. Solve for x and the measure of both angles.

$m\angle 1 = 12x+4$

$m\angle 2 = 9x+8$



$12x+4+9x+8=180$

$21x+\cancel{12}=180$

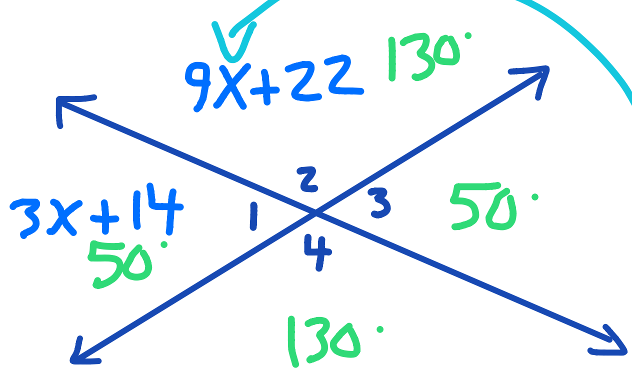
$\quad -\cancel{12} \quad -12$

$\frac{\cancel{21}x}{\cancel{21}} = \frac{168}{21}$

$\boxed{x=8}$

Ex.3 Find angle three and four if...

$m\angle 1 = 3x+14 \qquad m\angle 2 = 9x+22$



$9x+22+3x+14=180$

$12x+\cancel{36}=180$

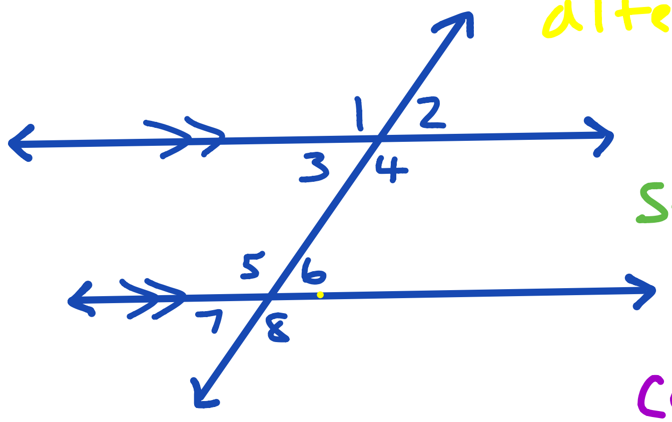
$\quad -\cancel{36} \quad -36$

$\frac{\cancel{12}x}{\cancel{12}} = \frac{144}{12}$

$\boxed{x=12}$

Lines and Transversals

- Transversal- is a line that intersects a system of two or more lines.



alternate exterior angles are congruent $\angle 2$ & $\angle 7$

same side exterior $\angle$'s are supplementary $\angle 2$ & $\angle 8$

corresponding $\angle$'s are $\cong$ $\angle 4$ & $\angle 8$

- Two line are parallel if they do not intersect.
- Perpendicular lines are two lines that intersect at a right angle.

- Corresponding angles- angles with the same relative position with spect to the transversal and the intersecting lines.
- Corresponding angles are congruent

$\angle 1 = \angle 5$

- Alternate interior angles- are on opposite sides of the transversal and lie on the interior of the two lines that the transversal intersect.

- Alternate interior angles are congruent.

$\angle 5 = \angle 4$

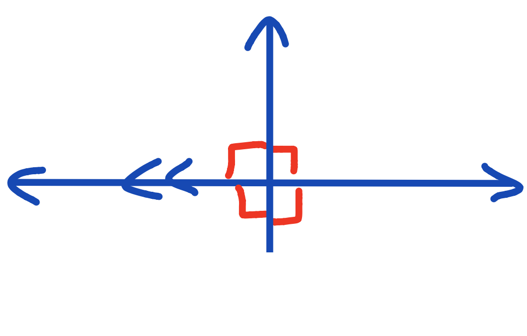
- Same side interior angles- are angles that lie on the same side of the transversal and are in between the lines that the transversal intersects.
- Same side interior angles are supplementary.

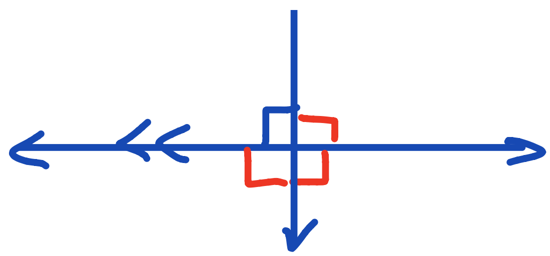
$\angle 6 + \angle 4 = 180^\circ$

- Alternate exterior angles- are angles that are on the opposite sides of the transversal and lie on the exterior of the two lines that the transversal intersects.
- Alternate exterior angles are congruent.

$\angle 1 = \angle 8$

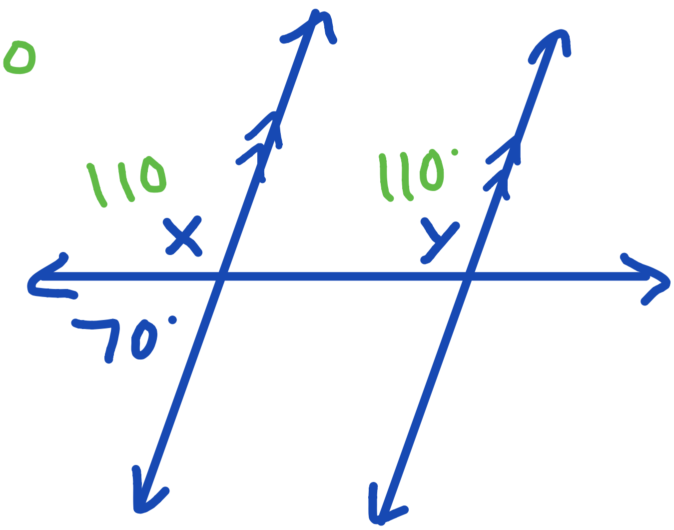
- Perpendicular Transversal Theorem- if a transversal is perpendicular to one of the two parallel lines, then it is perpendicular to the other.





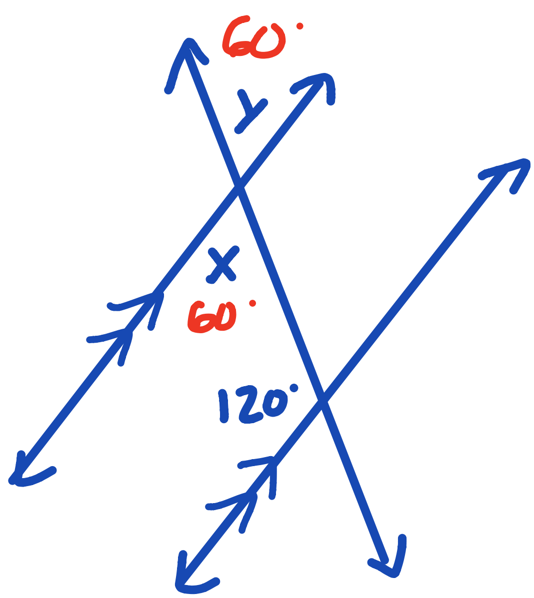
Ex.1

180-70

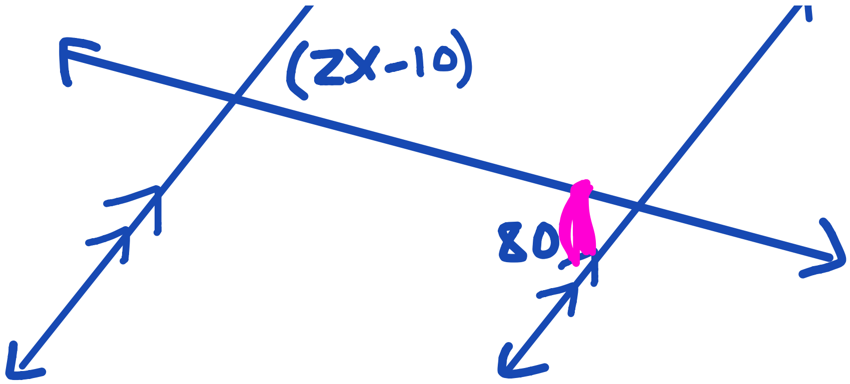


Ex.2

180-120



Ex.3

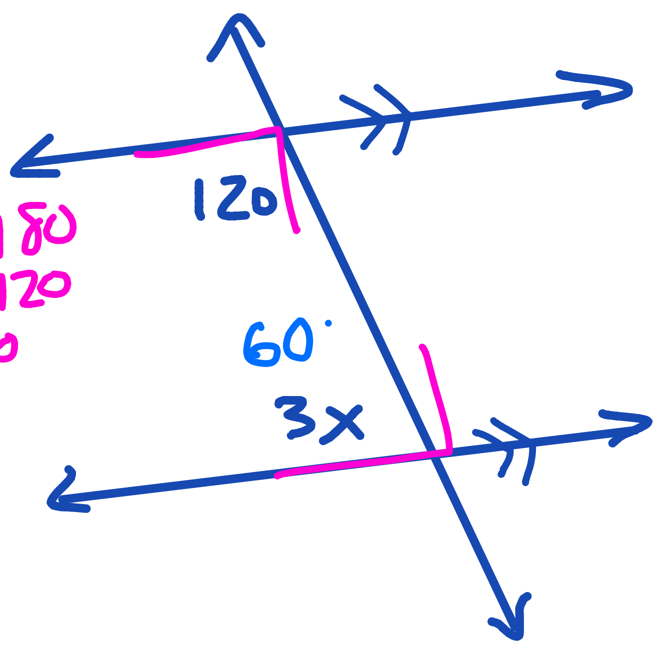


$$\cancel{80°} = 2x - \cancel{10}$$
$$+10 \qquad\qquad +\cancel{10}$$

$$\frac{90}{2} = \frac{\cancel{2}x}{\cancel{2}}$$

$$\boxed{x = 45°}$$

Ex. 4



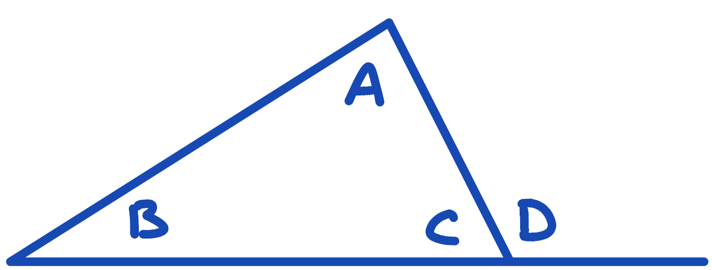
$$\cancel{120} + 3x = 180$$
$$-\cancel{120} \qquad\qquad -120$$

$$\frac{\cancel{3}x}{\cancel{3}} = \frac{60}{3}$$

$$\boxed{x = 20}$$

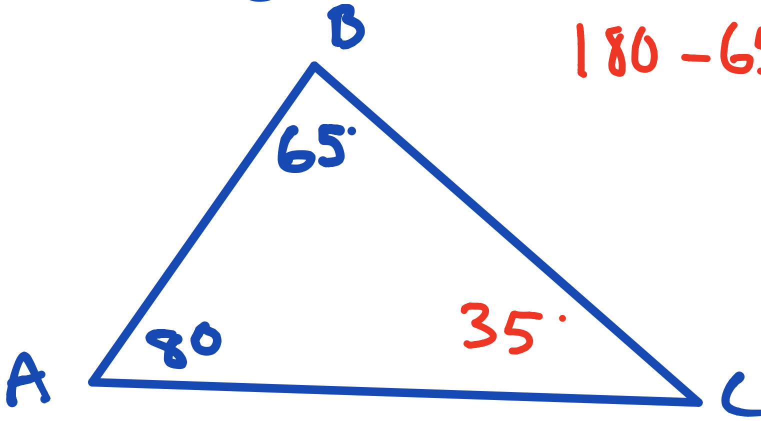
Proving Theorems about Triangles

- Triangle Sum Theorem- the sum of the angle measures of a triangle is 180 degrees.
- Scalene triangle- no congruent sides.
- Isosceles triangles- two congruent sides.
- Equilateral triangles- three congruent sides.
- Exterior Angle Theorem- the measure of an exterior angle of a triangle is equal to the sum of the measures of its remote interior angles.



$m\angle D = m\angle B + m\angle A$

Ex.1 Find angle C.

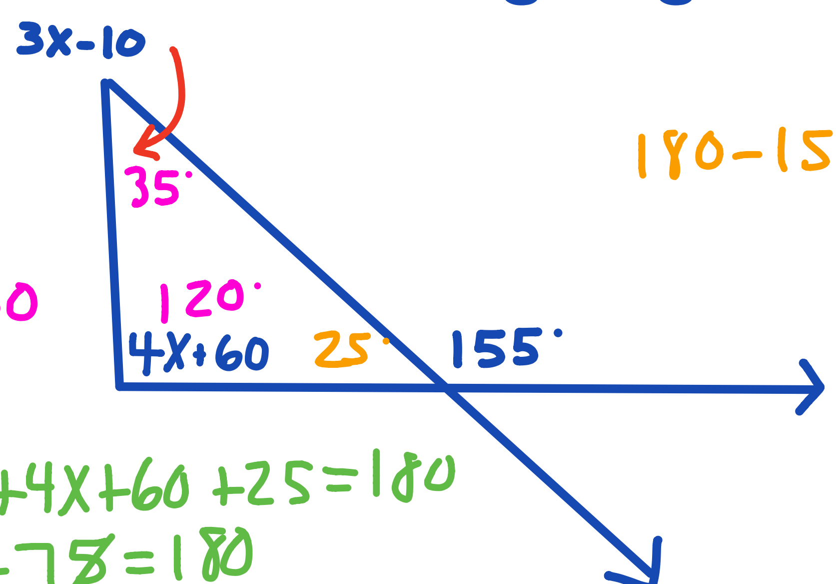


$180-65-80=35°$

Ex.2 Find the missing angles.

$180-155=25$

$4(15)+60$



$3x-10+4x+60+25=180$

$7x+75=180$

$-75 \quad -75$

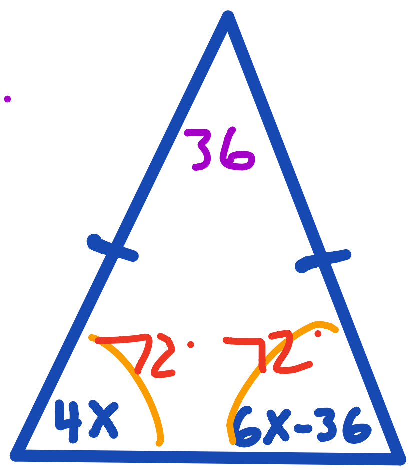
$\frac{7x}{7}=\frac{105}{7}$

$\boxed{x=15}$

- Equiangular- all angles are congruent.
- If a triangle is equilateral then it's equiangular.
- If a triangle is equiangular, then it is equilateral.

Ex.3 Find the measure of each angle.

$180-72-72°$

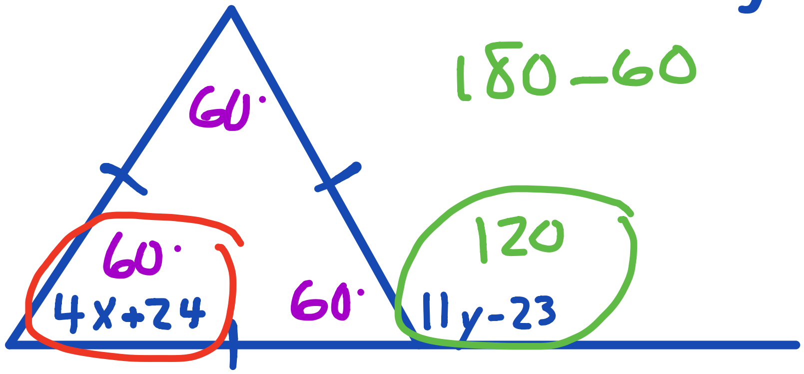


$$4x = 6x - 36$$
$$-6x \quad -6x$$
$$\frac{-2x}{-2} = \frac{-36}{-2}$$
$$\boxed{x = 18}$$

Ex.4 Find the values for x and y.

$\frac{180}{3} = 60°$

$180 - 60$



$4x + \cancel{24} = 60$
$-24 \quad -24$

$\frac{\cancel{4}x}{\cancel{4}} = \frac{36}{4}$

$\boxed{x = 9}$

$120 = 11y - \cancel{23}$
$+23 \qquad\qquad +23$

$\frac{143}{11} = \frac{\cancel{11}y}{\cancel{11}}$

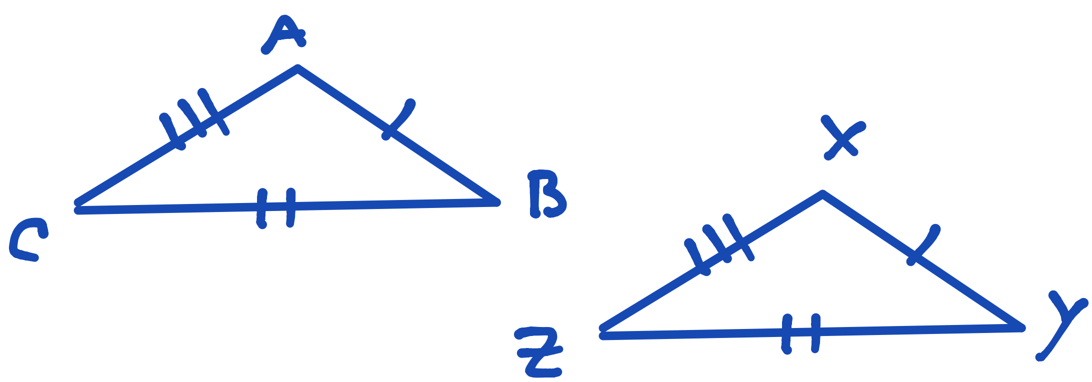
$\boxed{y = 13}$

Congruent Triangles

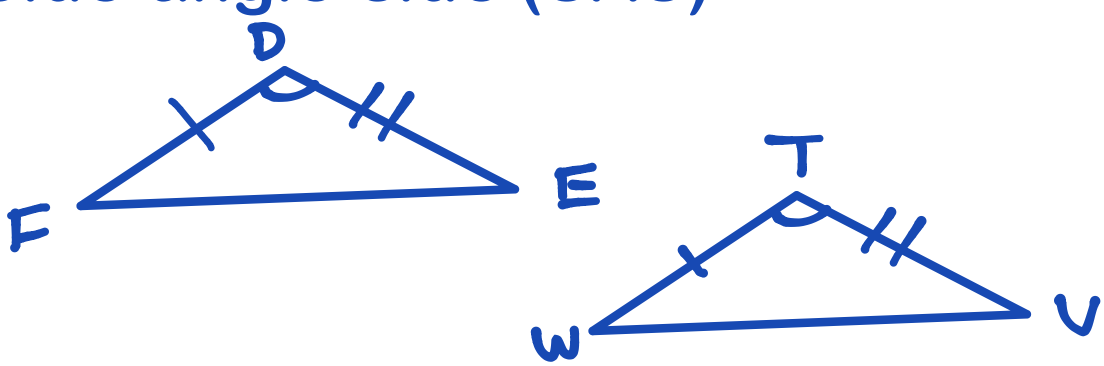
- If two or more triangles are proven congruent, then all of their corresponding parts are congruent.

Criteria for Congruence

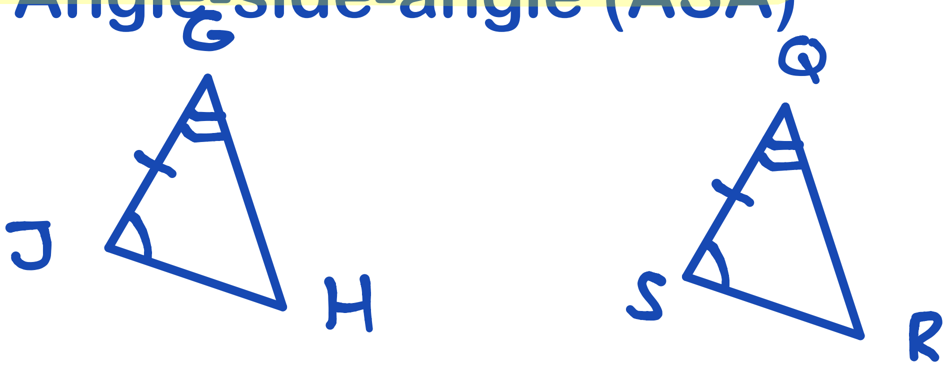
- side-side-side (SSS)



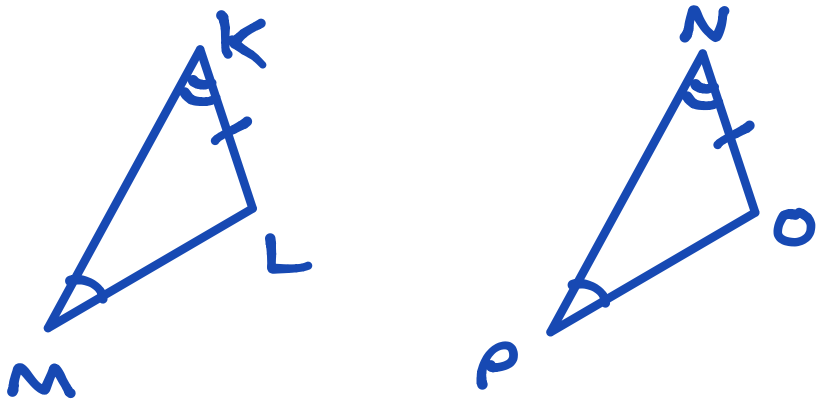
- Side-angle-side (SAS)



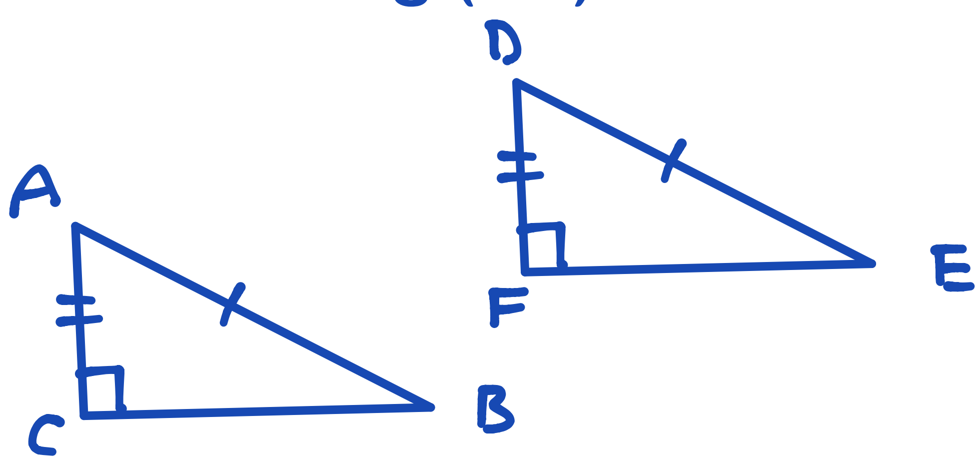
- Angle-side-angle (ASA)



- Angle-angle-side (AAS)

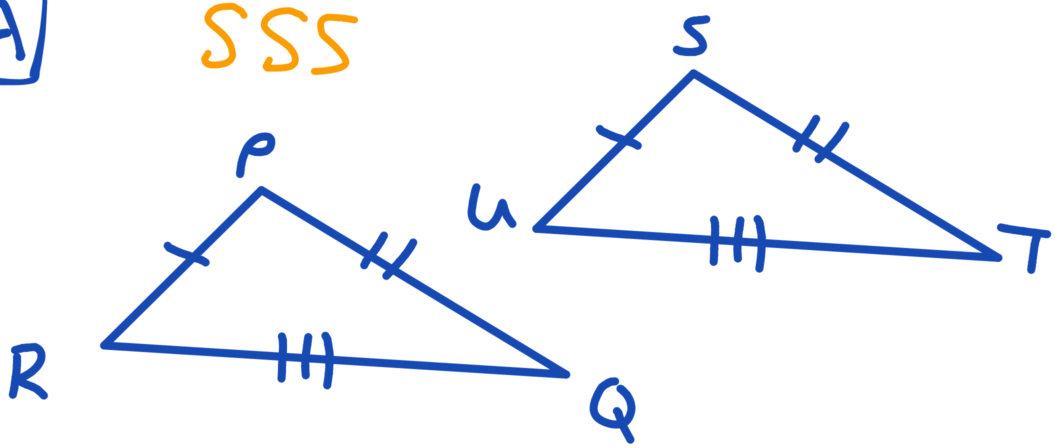


- Hypotenuse-Leg (HL)

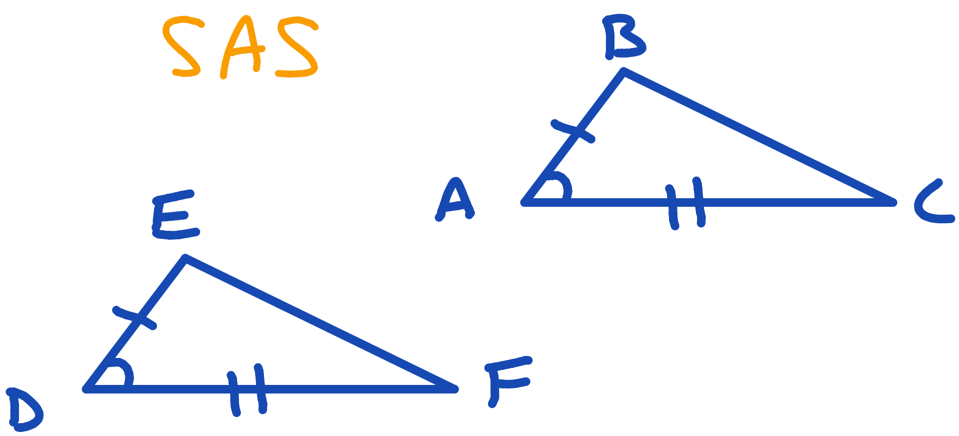


Ex.1 Determine which congruence statement can be used for the triangles.

$\boxed{A}$ SSS



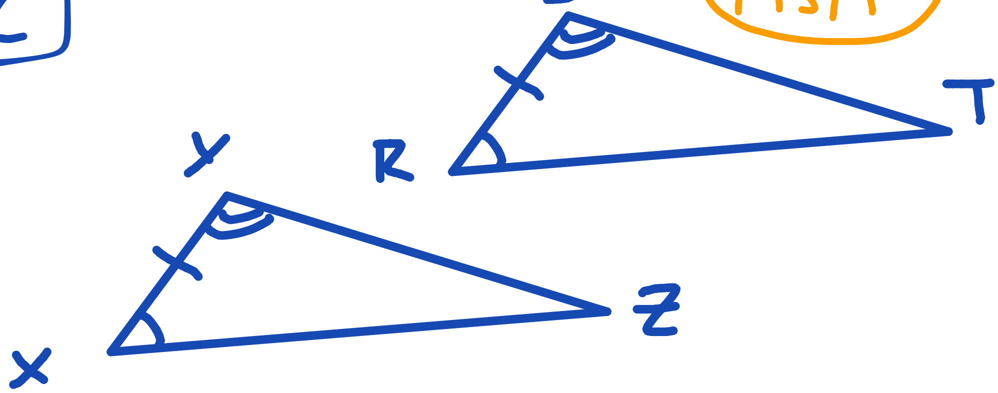
$\boxed{B}$ SAS



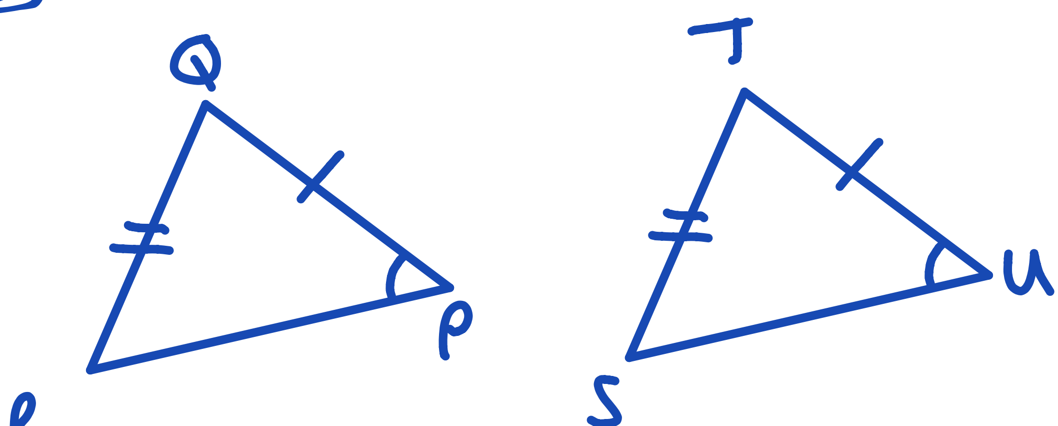
C

~~AAS~~

ASA



D



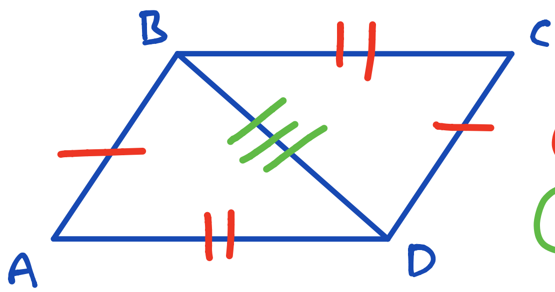
SSA ~~ASS~~

not enough information.

Two Column Proofs

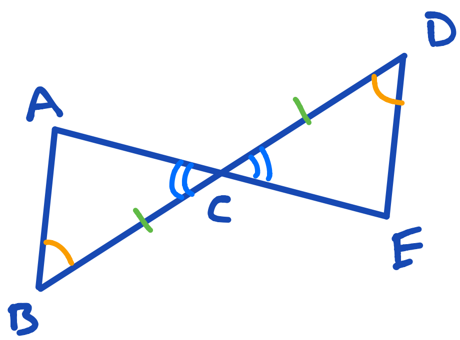
- Reflexive property: any quantity is equal to itself.
- Midpoint: a point that divides a segment into two congruent segments.
- Bisect: divide into two equal parts
- CPCTC: corresponding parts of corresponding triangles are congruent

Ex.1 Given: $\overline{AB} \cong \overline{CD}$, $\overline{AD} \cong \overline{CB}$



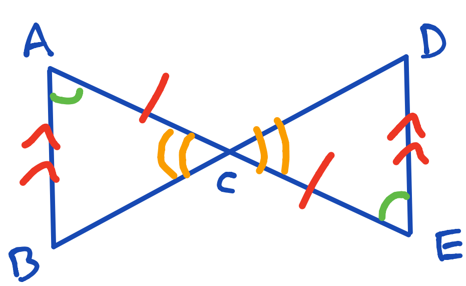
Statements	Reasons
① $\overline{AB} \cong \overline{CD}$	① Given
② $\overline{AD} \cong \overline{CB}$	② Given
③ $\overline{BD} \cong \overline{BD}$	③ Reflexive property
④ $\triangle ABD \cong \triangle CDB$	④ SSS

Ex.2 Given: $\overline{AE}$ bisects $\overline{BD}$, $\angle B \cong \angle D$



Statements	Reasons
① $\overline{AE}$ bisects $\overline{BD}$	① Given
② $\angle B \cong \angle D$	② Given
③ $\overline{BC} \cong \overline{CD}$	③ def. bisect
④ $\angle BCA \cong \angle DCE$	④ Vertical angles
⑤ $\triangle BCA \cong \triangle DCE$	⑤ ASA

Ex.3 Given: $\overline{AB} \parallel \overline{ED}$, $\overline{AC} \cong \overline{EC}$



Statements	Reasons
① $\overline{AB} \parallel \overline{ED}$	① Given
② $\overline{AC} \cong \overline{EC}$	② Given
③ $\angle A \cong \angle E$	③ Alternate interior ∠'s
④ $\angle BCA \cong \angle DCE$	④ Vertical Angles
⑤ $\triangle BCA \cong \triangle DCE$	⑤ ASA

12 SAS

14 $\triangle LMN$ $\triangle PQR$