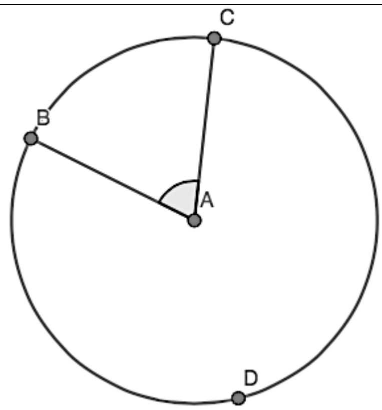
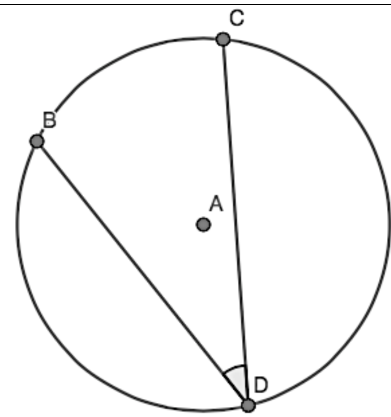
Angles of Circles Notes

Central Angle: An angle whose vertex is at the center of a circle.
Inscribed Angle: An angle whose vertex is on the circle and whose sides contain chords of a circle.
Arc measure: The angle that an arc makes at the center of the circle of which it is a part.
Chord: A segment whose endpoints are on a circle.



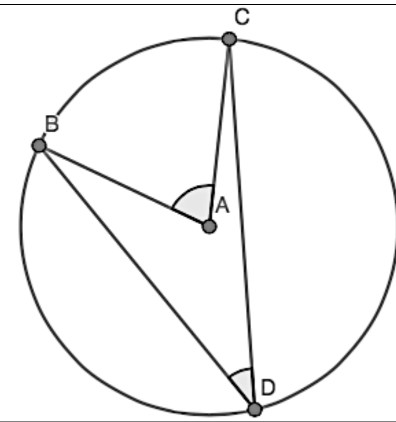
What can be said about $\angle BAC$?

∠A is a central angle.



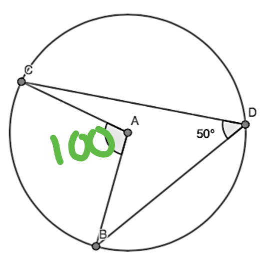
What can be said about $\angle BDC$?

∠D is an inscribed angle



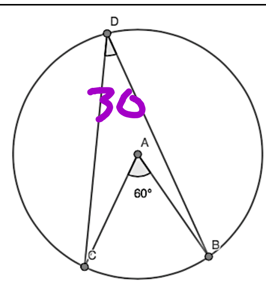
What can be said about $\angle BDC$ and $\angle BAC$?

∠A is a central angle. ∠D is a inscribed angle.



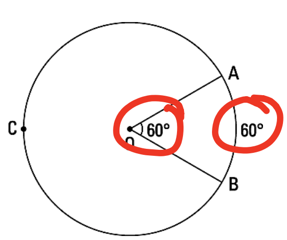
In words, how can you find $\angle CAB$? What is $\angle CAB$?

Multiply the inscribed angle by 2.



In words, how can you find $\angle CDB$? What is $\angle CDB$?

Divide the central angle by 2.

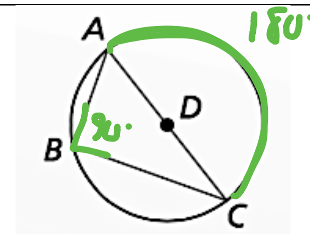
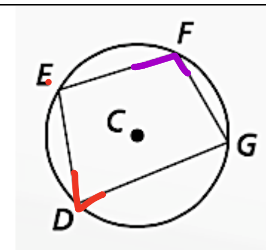
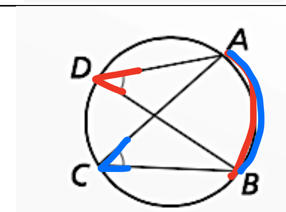


What can be said about arc measures and central angles?

Arc measures and central angles are the same.

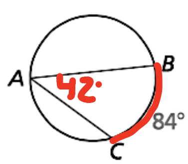
Inscribed Polygon Notes

Inscribed Polygon: A polygon whose vertices all lie on a circle.

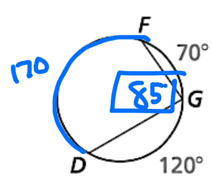
Inscribed Right Triangle Theorem If a right triangle is inscribed in a circle, then the hypotenuse is a diameter of the circle. Conversely, if one side of an inscribed triangle is a diameter of the circle, then the triangle is a right triangle and the angle opposite the diameter is the right angle. $m\angle ABC = 90^\circ$ if and only if $\overline{AC}$ is a diameter of the circle.	
Inscribed Quadrilateral Theorem A quadrilateral can be inscribed in a circle if and only if its opposite angles are supplementary. D, E, F, and G lie on $\odot C$ if and only if $m\angle D + m\angle F = m\angle E + m\angle G = 180^\circ$.	
Inscribed Angles of a Circle Theorem If two inscribed angles of a circle intercept the same arc, then the angles are congruent. $\angle ADB \cong \angle ACB$	

Examples

$m\angle A$

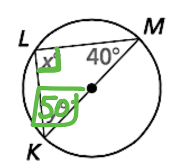


$m\angle G$



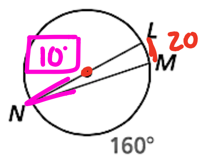
360-70-120

$\frac{170}{2} = 85^\circ$



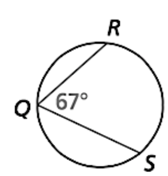
180-90-40

$m\angle N$

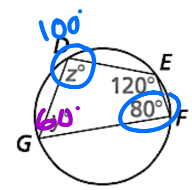


180-160=20

$m\widehat{RS}$



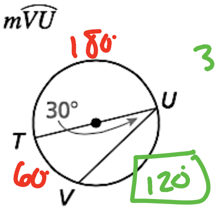
134



180-120=60

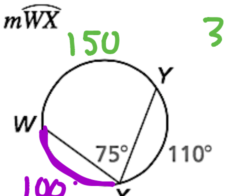
180-80=100

$m\widehat{VU}$

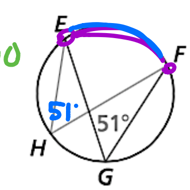


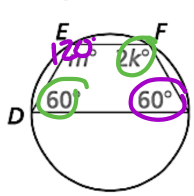
360-180-60=120

$m\widehat{WX}$



360-150-110=100





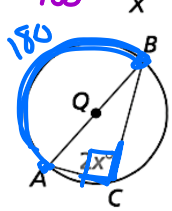
180-60=120

k=60

60+2k=180
-60 -60

$\frac{2k}{2} = \frac{120}{2}$

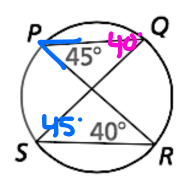
$\boxed{k=60}$



$\boxed{x=45^\circ}$

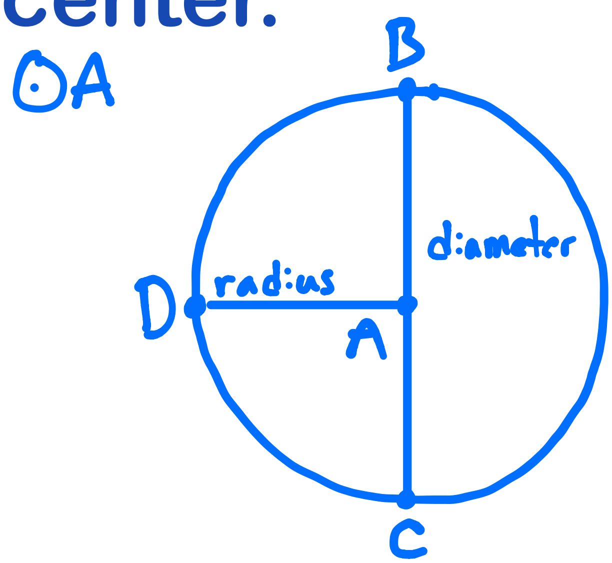
$\frac{2x}{2} = \frac{90}{2}$

$\boxed{x=45^\circ}$



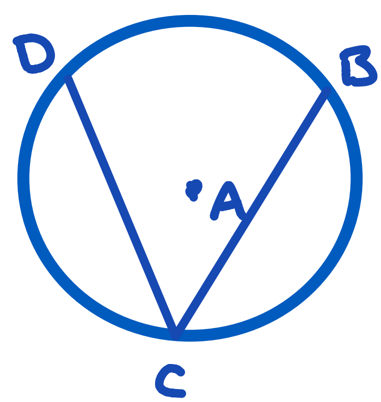
Circles

- Pi is the ratio of the circumference to the diameter of a circle.
- A circle is a set of all points that are equidistant from a fixed point.
- Concentric circles share the same center.



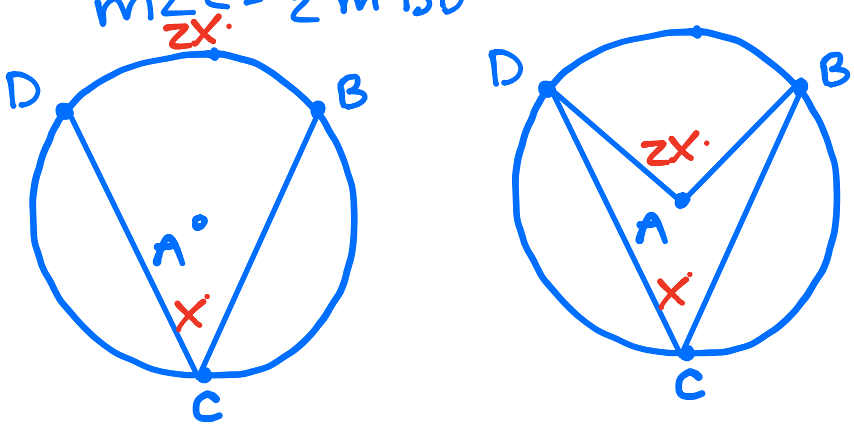
- Semicircle- is half a circle, 180 degrees.

- Major Arc- part of a circle that is larger than a semicircle.
- Minor arc- is a part of a circle that is smaller than a semicircle.
- Central angle- is an angle created with the vertex and two radii.
- Inscribed angle- formed when two cords whose vertex is on the circle.



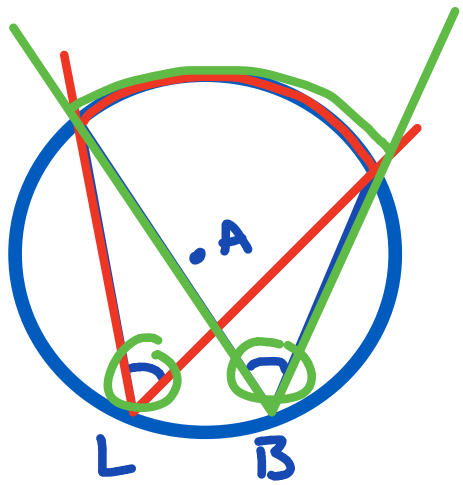
- **Intercepted arc- is an arc whose end points lie on the sides of an angle and whose other points are in the interior of the angle.**
- **Inscribed Angle Theorem: the measure of an inscribed angle is half the measure of its intercepted arcs angle.**

$m\angle C = \frac{1}{2} m\overset{\frown}{BD}$



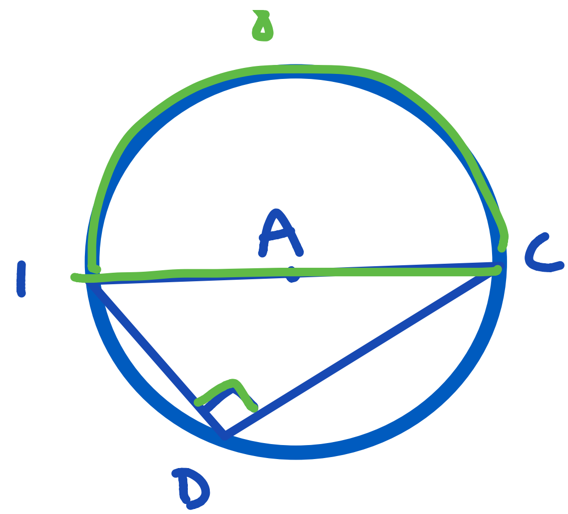
Corollary 1

- Two inscribed angles that intercept the same arc are congruent.



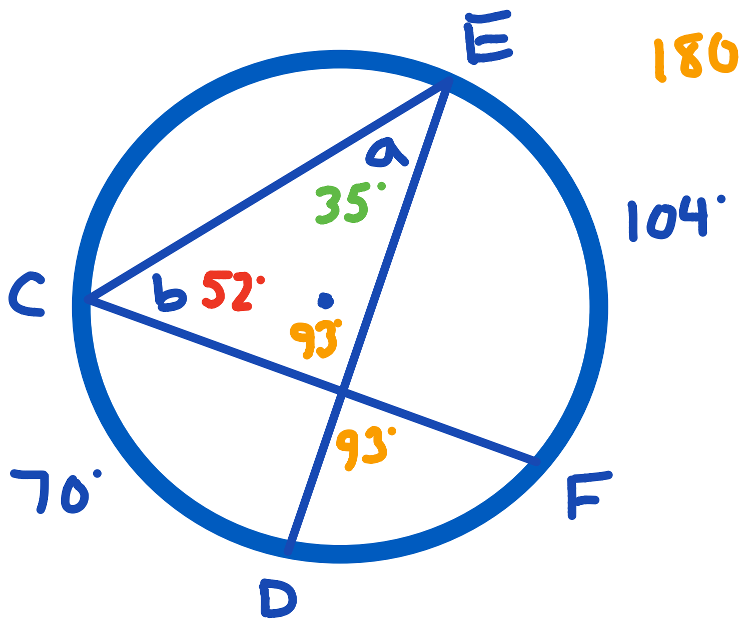
Corollary 2

- An angle inscribed in a semicircle is a right.



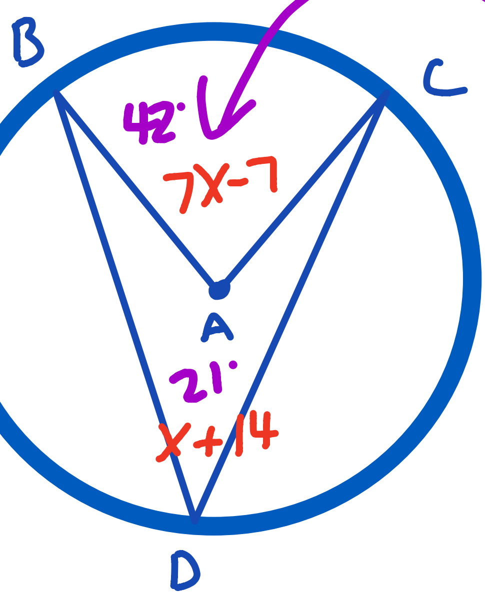
Ex.1 A car has a circular turning radius of 15.5 feet. The distance between the two front tires is 5.4 feet. To the nearest foot, how much farther does a tire on the outer edge of the turning radius travel than a tire on the inner edge if the car travels in one complete circle?

Ex.2 Find the value of each variable.



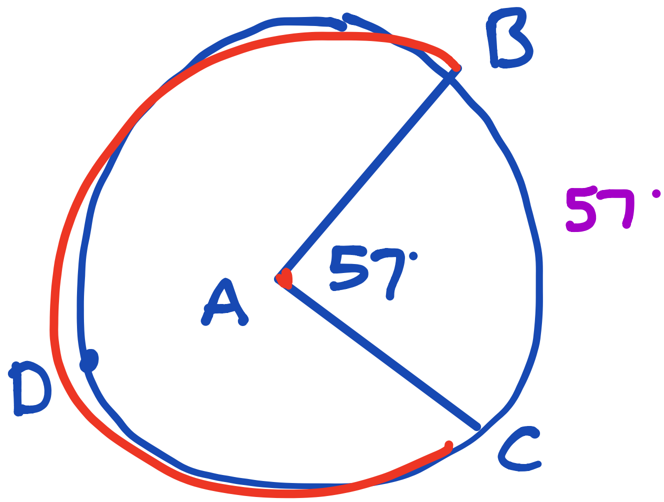
$180-52-35°$

Ex.3 Find the measure of angle BAC and BDC.



$$7x-7=2(x+14)$$
$$7x-7=2x+28$$
$$-2x+7 \quad -2x+7$$
$$\frac{5x}{5}=\frac{35}{5}$$
$$\boxed{x=7}$$

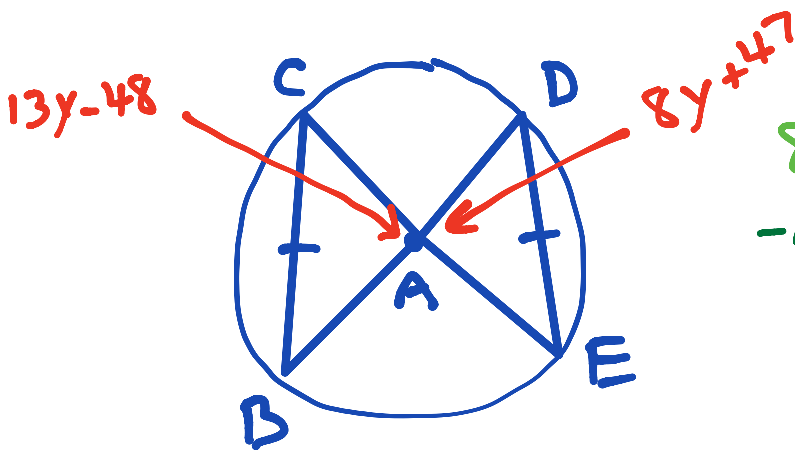
Ex.4 Find $\overset{\frown}{BDC}$.



$360 - 57 =$

$303°$

Ex.5 Find the value for y.



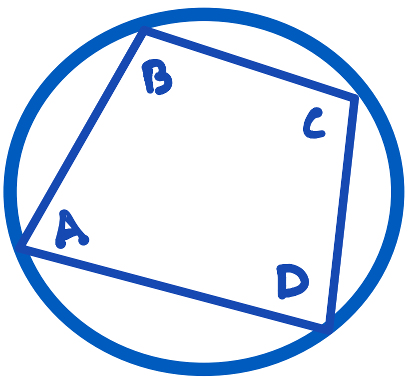
$$\cancel{8y} + 47 = 13y - 48$$
$$-\cancel{8y} + 48 \quad -8y + 48$$

$$\frac{95}{5} = \frac{\cancel{5}y}{\cancel{5}}$$

$$\boxed{y = 19}$$

Inscribed Quadrilaterals

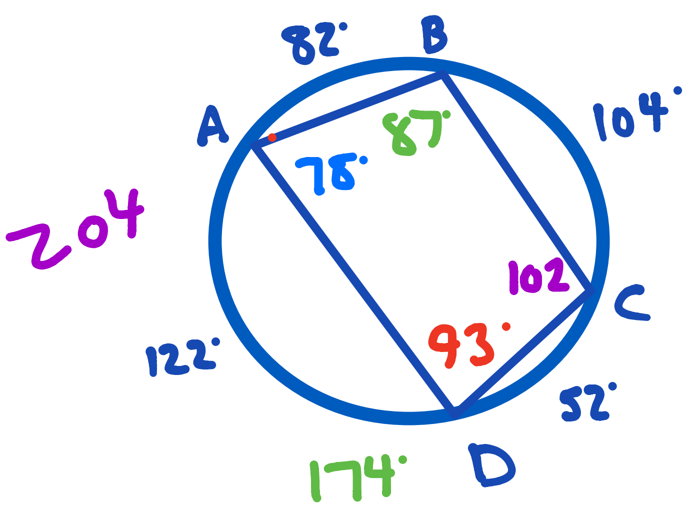
- Is a quadrilateral whose vertices are on a circle.
- Opposite angles of an inscribed quadrilateral are supplementary.



$m\angle A + m\angle C = 180^\circ$

$m\angle B + m\angle D = 180^\circ$

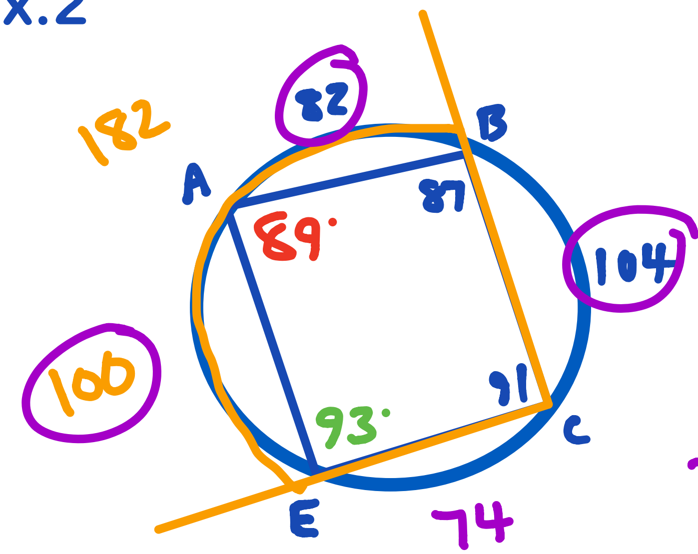
Ex.1 Find the missing angles.



$180 - 87 = 93^\circ$

$180 - 102 = 78^\circ$

Ex.2

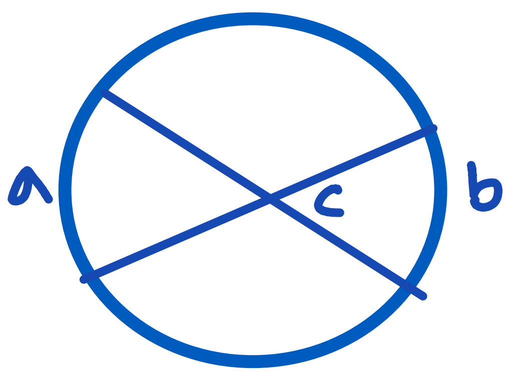


$180 - 91 = 89$

$180 - 87 = 93$

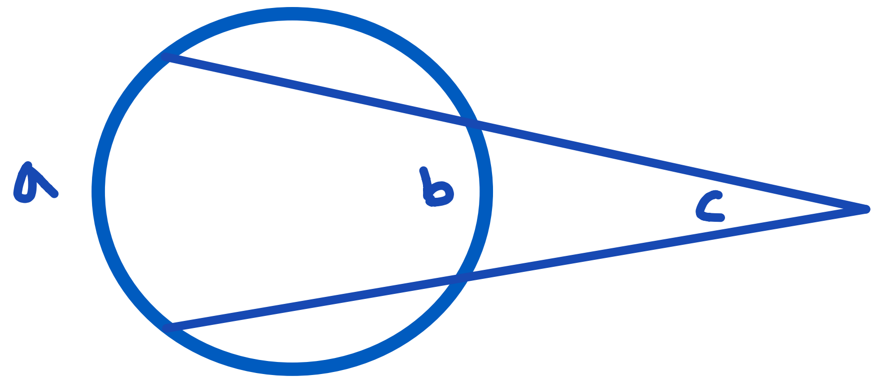
$360 - 100 - 104 - 82$

<u>Secant angle</u>



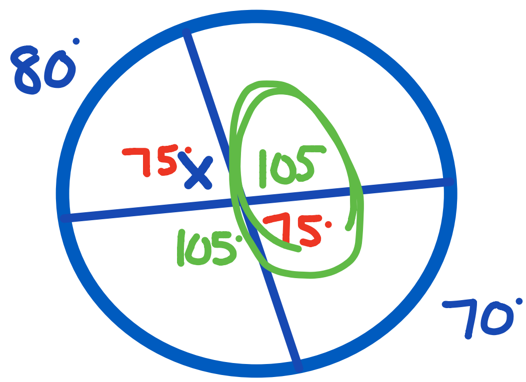
$$\frac{a+b}{2} = c$$

<u>**Circumscribed angle**</u> - the angle formed by two tangent lines whose vertex is outside of the circle is called the circumscribed angle.



$$\frac{a-b}{2}=c$$

Ex.1



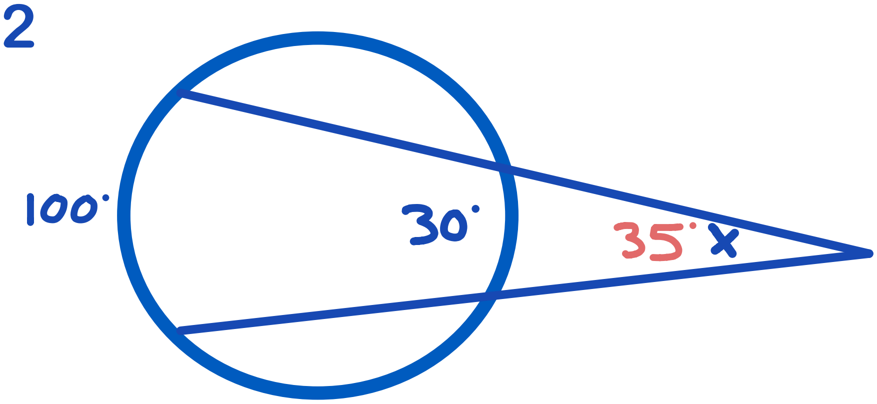
$$\frac{a+b}{2}=c$$

$$\frac{80+70}{2}=x$$

$$75^\circ=x$$

180-75

Ex.2

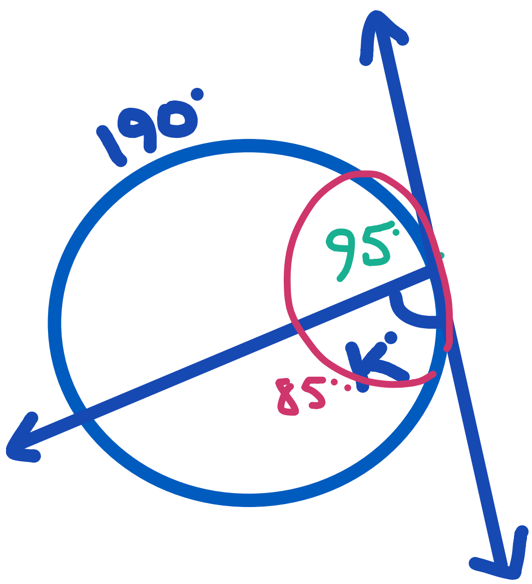


$$\frac{a-b}{2}=c$$

$$\frac{100-30}{2}=x$$

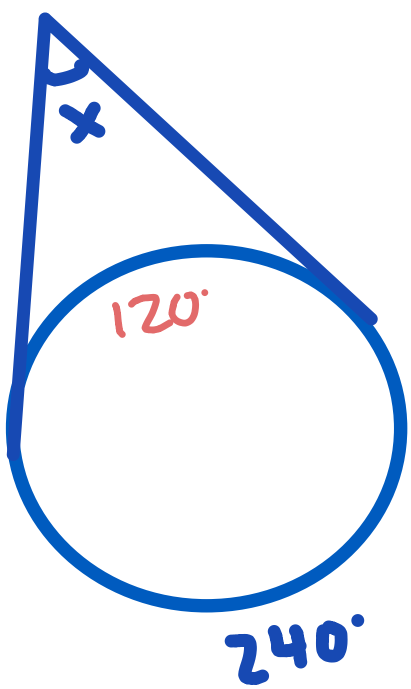
$$35^\circ=x$$

Ex.3



180−95

Ex.4



360−240=120

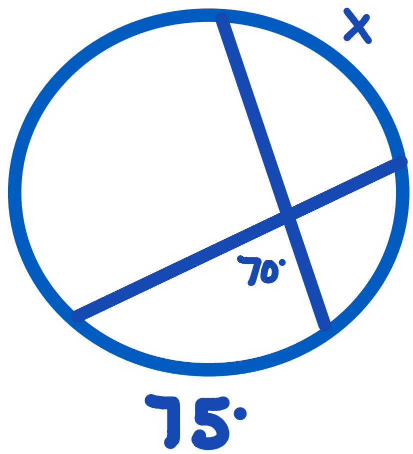
$$\frac{a-b}{2}=c$$

$$\frac{240-120}{2}=x$$

$$\frac{120}{2}=x$$

$$\boxed{60=x}$$

Ex.5



$$\frac{a+b}{2}=c$$

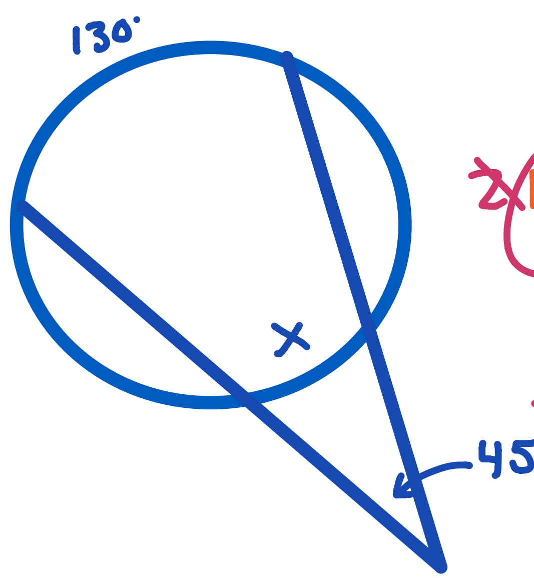
$$\cancel{2}\left(\frac{75+x}{\cancel{2}}\right)=(70)2$$

$$\cancel{75}+x=140$$

$$-\cancel{75}\qquad -75$$

$$\boxed{x=65^\circ}$$

Ex.6



$$\frac{a-b}{2}=c$$

$$\cancel{2}\left(\frac{130-x}{\cancel{2}}\right)=(45)2$$

$$\cancel{130}-x=90$$

$$-\cancel{130}\qquad -130$$

$$\frac{\cancel{-}x}{\cancel{-1}}=\frac{-40}{-1}$$

$$\boxed{x=40}$$

Radians

- Radian- a unit for measuring angles. One Radian is equal to length of the radius.

Ex.1 convert 40 degrees to radians.

$$\frac{40}{1} \cdot \frac{\pi}{180} = \frac{4\cancel{0}\pi}{18\cancel{0}} = \frac{4\pi}{18} = \boxed{\frac{2\pi}{9}}$$

Ex.2 convert $\frac{3\pi}{4}$ to degrees.

$$\frac{3\cancel{\pi}}{4} \cdot \frac{180}{\cancel{\pi}} - \frac{540°}{} = \boxed{135°}$$

Arc Length

- Circumference- the distance around a circle $C = 2\pi r$
- Arc length- portion of the circumference. $\text{Arc Length} = \frac{2\pi r \theta}{360^\circ}$
- Central angle- an angle with its vertex at the center of a circle.

Ex.1 What is the circumference of a circle that has an area of 1,000 m^2?

$A = 1000$

$A = \pi r^2$

$\frac{1000}{\pi} = \frac{\cancel{\pi} r^2}{\cancel{\pi}}$

$\sqrt{318} = \sqrt{r^2}$

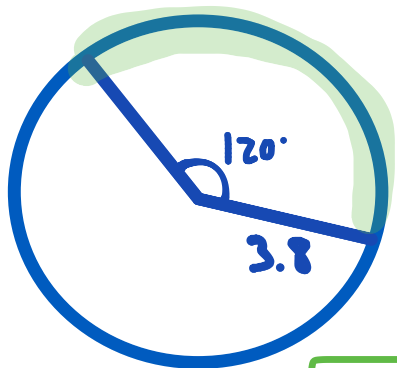
$\boxed{r = 17.8}$

$C = 2\pi r$

$C = 2\pi(17.8)$

$\boxed{C = 35.6\pi \approx 111.8}$

Ex.2 A circle has a radius of 3.8 units. Find the arc intercepted by a central angle measuring 120 degrees.



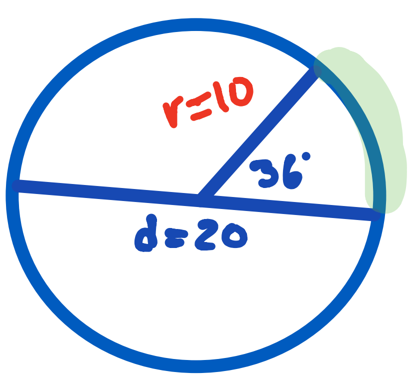
What is the arc length?

$$AL = \frac{2\pi r \theta}{360}$$

$$AL = \frac{2\pi(3.8)(120)}{360}$$

$$\boxed{AL = \frac{38\pi}{15} \approx 7.96}$$

Ex.3 a circle has a diameter of 20 feet. Find the length of an arc intercepted by a central angle measuring 36 degrees.



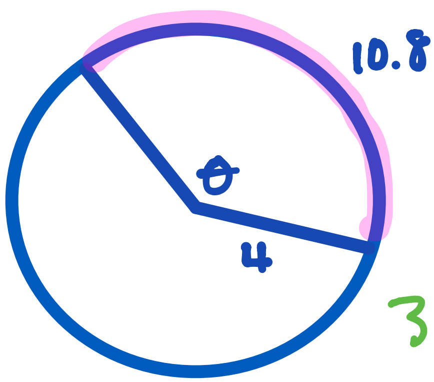
Find the arc length

$$AL = \frac{2\pi r \theta}{360°}$$

$$AL = \frac{2\pi(10)(36)}{360°}$$

$$\boxed{AL = 2\pi \approx 6.28}$$

Ex.4 a circle has a radius of 4 units. Find the central angle that intercepts an arc of length 10.8 units.



Find θ

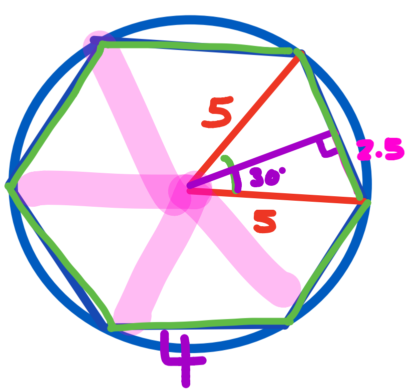
$$AL = \frac{2\pi r \theta}{360^\circ}$$

$$360(10.8) = \left(\frac{2\pi(4)\theta}{\cancel{360}}\right)\cancel{360}$$

$$\frac{3888}{(8\pi)} = \frac{\cancel{2\pi 4}\,\theta}{\cancel{8\pi}}$$

$$\boxed{\theta = 155^\circ}$$

Ex.1 Show how the perimeter of a hexagon can be used to find an estimate for the circumference a circle that has a radius of 5 meters.



$c = 2\pi r$

$c = 2\pi(5)$

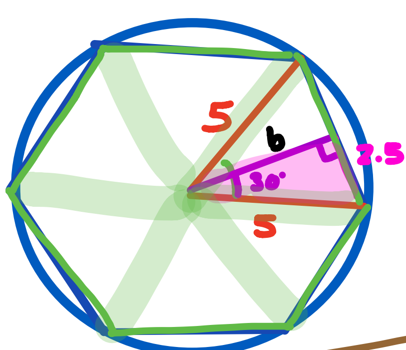
$\boxed{c = 31}$

$\frac{360^\circ}{6} = 60$

$P = 12(2.5)$

$\boxed{P = 30}$

Ex.2 show how the area of a hexagon can be used to find an estimate for the area of a circle that has a radius of 5 meters.



$A = \pi r^2$

$= \pi(5)^2$

$\boxed{= 78.5m^2}$

$A = \frac{1}{2}bh$

$= \frac{1}{2}(2.5)(4.3)$

$= 5.4$

$12(5.4) = \boxed{64.8m^2}$

$a^2 + b^2 = c^2$

$2.5^2 + b^2 = 5^2$

$-2.5^2 \qquad -2.5^2$

$\sqrt{b^2} = \sqrt{18.75} \qquad b = 4.3$

Areas of Sectors

- Area of a circle- $A = \pi r^2$
- A sector is a portion of a circle bounded by two radii and their intercepted arc.

$$\text{Area Sector} = \frac{\pi r^2 \theta}{360} \leftarrow \text{degrees}$$

$$\text{Area Sector} = \frac{\pi r^2 \theta}{2\pi} \leftarrow \text{radians}$$

Ex.1 Find the area of a circle that has a circumference of 100 meters.

$C = 100$

$C = 2\pi r$

$\frac{100}{(2\pi)} = \frac{\cancel{2\pi} r}{\cancel{2\pi}}$

$\boxed{r = 15.9}$

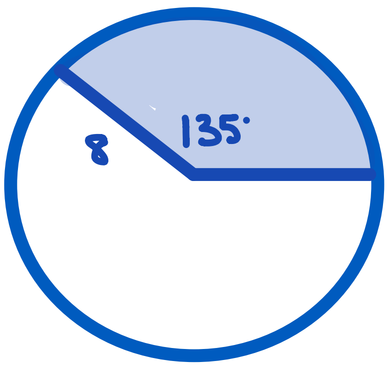
$A = \pi r^2$

$A = \pi (15.9)^2$

$\boxed{A = 252.8\pi \approx 794}$

Ex.2 a circle has a radius of 8 units. Find the area of a sector with a central angle of 135.

Find the area of the sector.

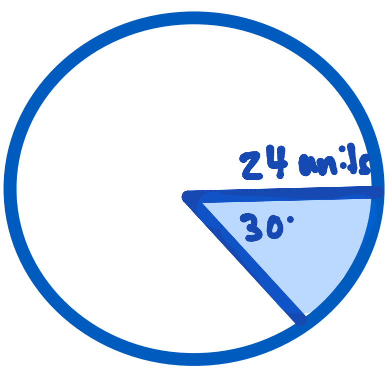


$$AS = \frac{\pi r^2 \theta}{360°}$$

$$AS = \frac{\pi (8)^2 (135)}{360}$$

$$\boxed{AS = 24\pi \approx 75.4}$$

Ex.3 A circle has a radius of 24 units. Find the area of a sector with a central angle of 30 degrees.



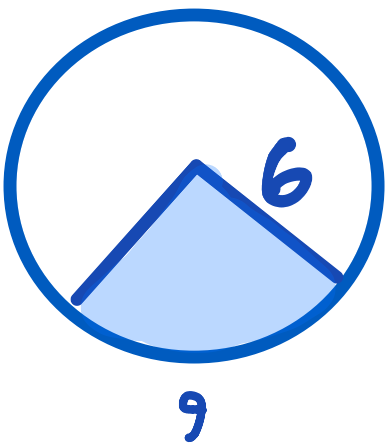
$$AS = \frac{\pi r^2 \theta}{360}$$

$$AS = \frac{\pi (24)^2 (30)}{360°}$$

$$\boxed{AS = 48\pi \approx 150.8}$$

Ex.4 a circle has a radius of 6 units. Find the area of a sector with an arc length of 9 units.

Find θ



$$AL = \frac{2\pi r\theta}{360}$$

$$9 = \left(\frac{2\pi(6)\theta}{\cancel{360}}\right)3\cancel{60}$$

$$\frac{3240}{12\pi} = \frac{\cancel{2\pi 6}\theta}{\cancel{12\pi}}$$

$$\boxed{\theta = 86}$$

$$AS = \frac{\pi r^2\theta}{360} = \frac{\pi(6)^2(86)}{360} = \boxed{\frac{43\pi}{5} \approx 27}$$

$$\boxed{AS = 8.6\pi = \frac{43\pi}{5} = 27u^2}$$