Probability

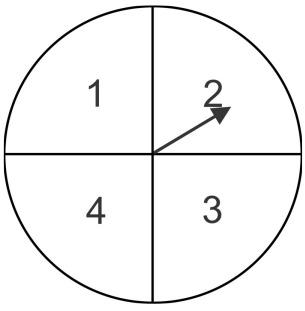
Set - a set is a collection of objects
Subset - part of a larger set $A \subset B$
Element - any object of a set
Empty set - the set that has no elements
Union- elements that are in both sets. or $A \cup B$
Intersection- what sets share in common.

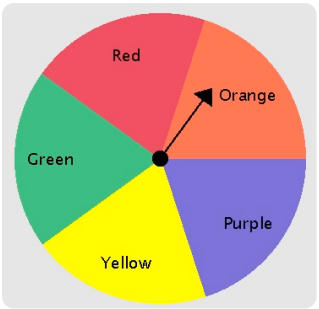
and $A \cap B$

Complement- is the set of elements that are not in the set. $\overline{A}$ A'

Ex.1 List the sample space for the spinners below using set notation.

$A = \{1, 2, 3, 4\}$





$B = \{\text{red, green, purple, orange, yellow}\}$

M = {1,2,3,4,5,6,7}
N = {1,3,5,7}

Ex.2 what is M ∩ N.

{1,3,5,7}

Ex.3 what is M ∪ N.

{1,2,3,4,5,6,7}

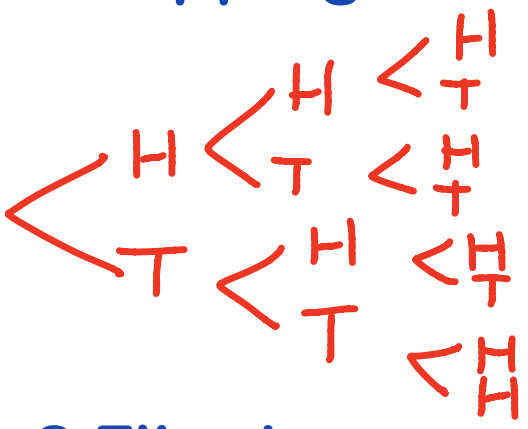
Ex.4 is N a subset of M?

yes because every element in N is also in M.

<u>The fundamental counting principle.</u>
If you have (a) ways of doing event 1, (b) ways of doing event 2, and (c) ways of doing event 3, then you can find the total number of outcomes by multiplying.

a x b x c

Ex.1 Flipping a coin 3 times.



HHH THH
HHT THT
HTH TTH
HTT TTT

$2 \cdot 2 \cdot 2 = \boxed{8}$

Ex.2 Flipping a coin and rolling a dice.

$2 \cdot 6 = \boxed{12}$

Ex.3 Three choices of sandwiches, three choices of sides, and 2 choices of drinks.

$3 \cdot 3 \cdot 2 = \boxed{18}$

Probability

- Is a number from 0 to 1 or a percent from 0% to 100%
- that indicates how likely an event is to occur.

Probability of an Event

- When all the outcomes are equally likely, the probability of an event E:

$$P(E) = \frac{\text{number of outcomes}}{\text{number of outcomes in the sample space}}$$

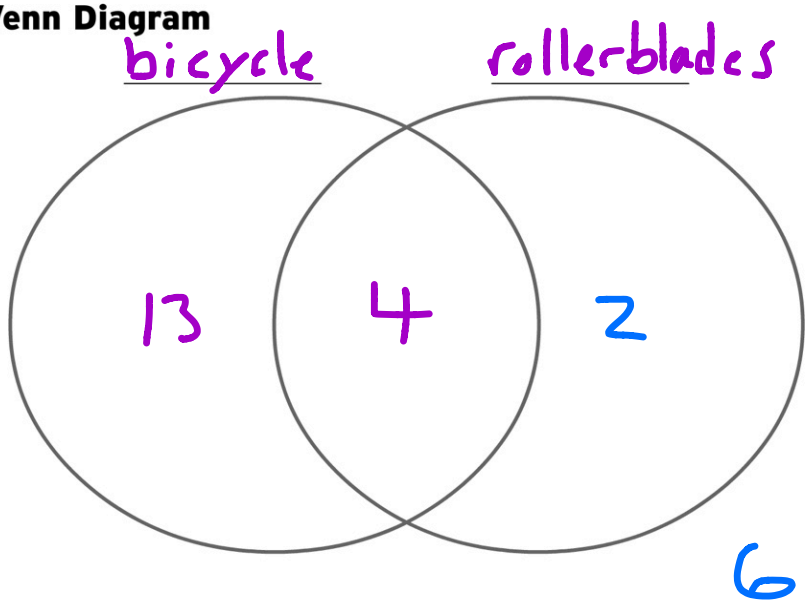
Venn Diagram

- Can be used to express the relationships between various sets.

Ex.1 There are 25 students in a geometry class. 17 students own a bicycle and 6 students own rollerblades. 4 students own both.

create a Venn Diagram.

Venn Diagram



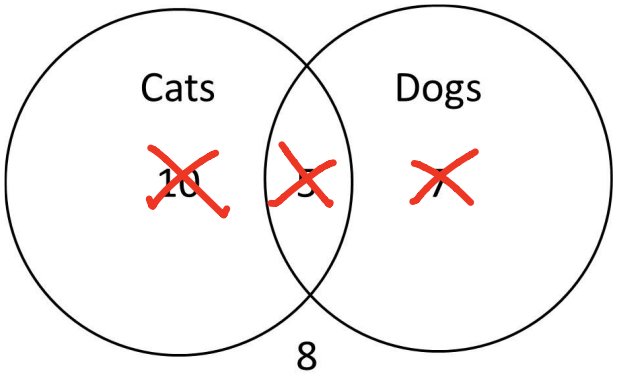
Ex.2 Find the probability a student owns a cat and a dog. $\frac{5}{30} = \frac{1}{6}$

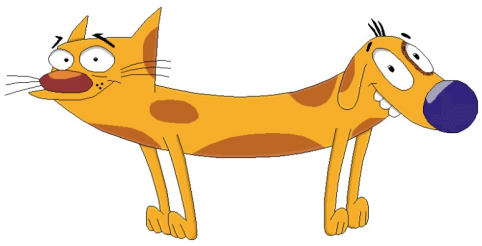
Ex.3 find P(C ∪ D) $\frac{22}{30}$

Ex.4 find P(C) $\frac{15}{30}$

Ex.5 P(C)' $\frac{15}{30}$

Ex.5 P($\overline{C \cup D}$) $\frac{8}{30}$





Student	dog	cat	hamster	bird	fish
1					
2	✓	✓			
3	✓				
4	✓				
5			✓		✓
6					✓
7	✓	✓		✓	
8					
9		✓		✓	
10	✓	✓			

Ex.1 Find probability of having a dog and cat.

$$P(D \cap C) = \frac{3}{10} = .3 = 30\%$$

Ex.2 Find the probability of having a hamster or fish.

$$P(H \cup F) = \frac{2}{10} = .2 = 20\%$$

Ex.3 what's the probability of not having a bird?

$P(\overline{B}) = \frac{8}{10} = .8 = 80\%$

$P(B)' =$

Ex.4 Find the probability of not having a dog and cat.

$P(\overline{D \cap C}) = \frac{7}{10} = .7 = 70\%$

Ex.5 Find the probability of not having a dog or cat.

$P(\overline{D \cup C}) = \frac{4}{10} = .4 = 40\%$

Mutually Exclusive vs. Overlapping

- Compound- two or more events
- Mutually exclusive- no common outcomes
- Overlapping- common outcomes.

Mutually exclusive-

- the probability that one or the other of several events will occur is found by summing the individual probabilities of the events.
- P(A or B)= P(A) + P(B)

Ex.1 Find the probability that a girls favorite department store is Macys or Nordstrom.

0.25+ 0.20 = .45 45%

Find the probability that a girls favorite store is not JC Penny's.

.25+.20+.20+.25 = .90

Macy's	0.25
Sake Fifth Ave	0.20
Nordstrom	0.20
JC Penny's	0.10
Bloomingdale's	0.25

Ex.2 When rolling two dice, what is the probability that the sum will be 4 or 5?

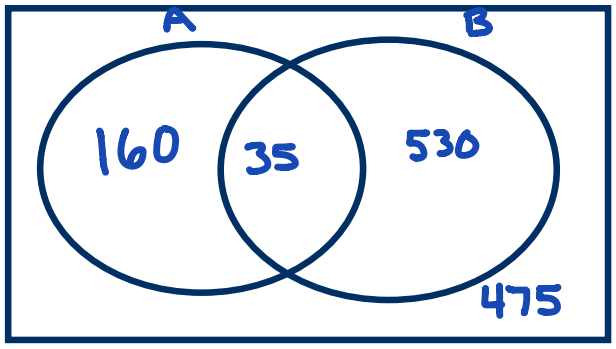
	1	2	3	4	5	6
1	2	3	4	5	6	7
2	3	4	5	6	7	8
3	4	5	6	7	8	9
4	5	6	7	8	9	10
5	6	7	8	9	10	11
6	7	8	9	10	11	12

$\frac{3}{36} + \frac{4}{36} = \boxed{\frac{7}{36}}$

Overlapping Events

- probability that non-mutually exclusive events A and B or both will occur expressed as:
- P(A or B) = P(A) + P(B) - P(A and B)

Ex.3 Find P(A or B).



Ex.4 Find the probability of picking a king or a club in a deck of cards.

$$\frac{4}{52} + \frac{13}{52} - \frac{1}{52} = \boxed{\frac{4}{13}}$$

Ex.5 Find the probability of picking a female or a person from Florida out of the committee

	Female	Male	
Florida	8	4	12
Alabama	6	3	9
Georgia	7	3	10
	21	10	31

$$\frac{21}{31} + \frac{12}{31} - \frac{8}{31} = \boxed{\frac{25}{31}}$$

Conditional Probability

- contains a condition that limits the sample space of an event.
- written as, P(A|B) the probability of event A, given event B.
- $P(A|B) = \frac{P(A \cap B)}{P(B)}$

	Blue eyes	Brown eyes	Total
Female	8	6	14
Male	5	7	12
Total	13	13	26

Ex.1 Find P(blue eyes | female) $\frac{8}{14}$

Ex.2 Find P(female | blue eyes)

$\frac{8}{13}$

Ex.3 Find P(male | blue eyes)

$\frac{5}{13}$

Ex.4 Find probability they have brown eyes, given they are female. $\frac{6}{14}$

Independent Events

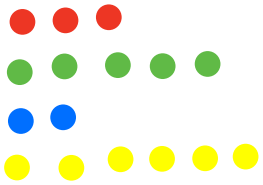
- Event A occurring does NOT affect the probability of event B occurring.
- P(A and B) = P(A) P(B)

Ex.1 A coin is tossed and a 6-sided die is rolled. Find the probability of landing on the head side of the coin and rolling a 3 on the die.

H T 1 2 3 4 5 6

$\frac{1}{2} \cdot \frac{1}{6} = \boxed{\frac{1}{12}}$

Ex.2 A jar contains 3 red, 5 green, 2 blue, and 6 yellow marbles. What is the probability of choosing a green and a yellow marble?



$\frac{5}{16} \cdot \frac{6}{16} = \frac{30}{256} = \boxed{\frac{15}{128}}$

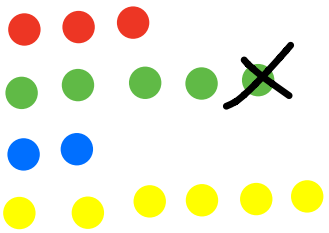
Ex.3 A school a survey found that 9 out of 10 students like pizza. If three are chosen at random with replacement, what is the probability that all three students like pizza?

$\frac{9}{10} \cdot \frac{9}{10} \cdot \frac{9}{10} = \boxed{\frac{729}{1000}}$

Dependent Events

- Event A occurring affects the probability of event B occurring.
- Key words, without replacing
- P(A and B) = P(A) P(B|A)

Ex.4 A jar contains 3 red, 5 green, 2 blue, and 6 yellow marbles. A marble is chosen at random from the jar. A second marble is chosen without replacing the first one. What is the probability of choosing a green and yellow marble?



$\frac{5}{16} \cdot \frac{6}{15} = \frac{30}{240} = \boxed{\frac{1}{8}}$

Ex.5 An aquarium contains 6 male goldfish and 4 female goldfish. You randomly select a fish from the tank, do not replace it, and then randomly select a second fish. What is the probability that both are male?

MMMMMM
FFFF

$\frac{6}{10} \cdot \frac{5}{9} = \frac{30}{90} = \boxed{\frac{1}{3}}$

$$P(A \cap B) = P(A) \cdot P(B)$$

$$.3 = (.6)(.5)$$

$$.3 = .3 \checkmark$$

#8

$P(M) = .6$

$P(S) = .5$

$P(M \cap S) = .3$

yes, they are independent events.

#9

$P(F) = 60\%$

$P(LH) = 50\%$

$P(F \cap LH) = 45\%$

$P(LH \mid F) = 75\%$

$P(A \cap B) = P(A)P(B)$

$45\% = 60\% \cdot 50\%$

$45\% \neq 30\%$

dependent

Using probability formulas working backwards

- **Substitute in the information you have in the appropriate formula, then solve for the missing piece.**

Multiplication rule: Independent

$$P(A \cap B) = P(A) \cdot P(B)$$

Addition rule: overlapping

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

Conditional Probability: dependent

$$P(A \cap B) = P(A) \cdot P(B|A)$$

Ex.1 the probability of Sam getting an A on the Chemistry test is 0.76. The probability of him getting an A on his Calculus test and an A on his Chemistry test is 0.494. What is the probability of him getting an A on his calculus test given that he got an A on his chemistry test?

Ex.2 An optional camp to improve players basketball skills was held in the county. The probability of a kid attending was 0.62. The probability that they attended and made the honor roll was 0.44. What is the probability that they made the honor roll?

Ex.3 P(A)=1/4, P(B)=5/8, P(A U B)= 3/4, find P(A and B).

Name: _____ Date: _____

Compound Probability: Mutually Exclusive vs. Overlapping

UNIT QUESTION: How do you use probability to make plans and predict for the future? (Standard: MM1D1-3)

Today's Question: When do I add or multiply when solving compound probabilities? (Standard: MM1D2.a,b.)

Vocabulary:

- ☆ Compound Event
  - combines two or more events
- ☆ Mutually Exclusive
  - no common outcomes
- ☆ Overlapping
  - have at least one common outcome

or

Mutually Exclusive

The probability that one or the other of several events will occur is found by summing the individual probabilities of the events:

$$P(A \text{ or } B) = P(A) + P(B)$$

1. Find the probability that a girl's favorite department store is Macy's or Nordstrom.

   .25 + .20 = .45

   Find the probability that a girl's favorite store is not JC Penny's.

   .25 + .20 + .20 + .25 = .90

Macy's	0.25
Saks Fifth Ave.	0.20
Nordstrom	0.20
JC Penny's	~~0.10~~
Bloomingdale's	0.25

2. When rolling two dice, what is probability that your sum will be 4 or 5?

$$\frac{3}{36} + \frac{4}{36} = \frac{7}{36}$$

+	1	2	3	4	5	6
1	2	3	4	5	6	7
2	3	4	5	6	7	8
3	4	5	6	7	8	9
4	5	6	7	8	9	10
5	6	7	8	9	10	11
6	7	8	9	10	11	12

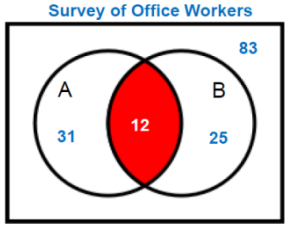
~~3.~~ What is the probability of picking a queen or an ace from a deck of cards?

Overlapping Events

Probability that non-mutually exclusive events A and B or both will occur expressed as: P(A or B)

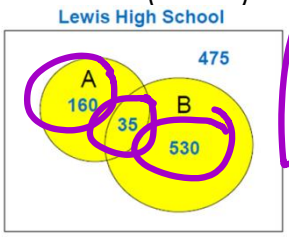
$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

4. Find the probability that a person will drink both.



$\frac{12}{151}$

5. Find the $P(A \cup B)$.



$\frac{725}{1200}$

6. Find the probability of picking a king or a club in a deck of cards.

7. Find the probability of picking a female or a person from Florida out of the committee members.

$\frac{21}{31} + \frac{12}{31} - \frac{8}{31} = \frac{25}{31}$

	Female	Male	
Florida	8	4	12
Alabama	6	3	9
Georgia	7	3	10
	21	10	31

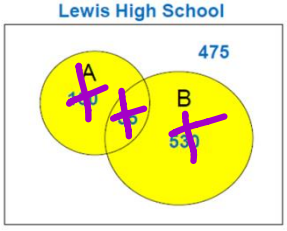
8. When rolling 2 dice, what is the probability of getting an even sum or a number greater than 10?

$\frac{18}{36} + \frac{3}{36} - \frac{1}{36} = \frac{20}{36}$

+	1	2	3	4	5	6
1	2	3	4	5	6	7
2	3	4	5	6	7	8
3	4	5	6	7	8	9
4	5	6	7	8	9	10
5	6	7	8	9	10	11
6	7	8	9	10	11	12

9. Find the $P(\overline{A \cup B})$.

$\frac{475}{1200}$



10. Find the $P(A)'$.

$\frac{428}{454}$