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Worksheet 5-1: Periodic Functions and Their Properties

A function that produces a graph that has a regular repeating pattern over a constant interval is called a **periodic function**. It describes something that happens in a **cycle, repeating in the same way** over and over.

Properties of Periodic Function:

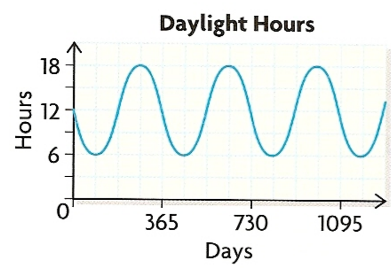
- its graph repeats at regular intervals
- its *y*-values in the table of values show a repetitive pattern when *x*-values change by the same increment
- its cycle is one repetition of a periodic pattern
- its period is the horizontal length of a cycle on a graph. The period can be in units of time or other units of measurement.
- its amplitude is half the difference between the maximum value (peak) and the minimum value (trough) in a cycle.
- its equation of the axis is the equation of the horizontal line halfway between the maximum and the minimum values.

$$\text{Amplitude} = \frac{\max - \min}{2}$$

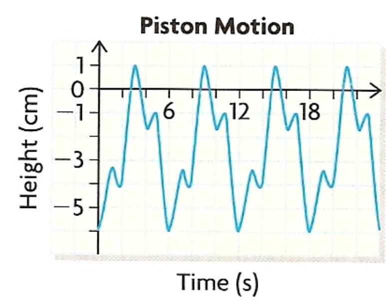
$$y = \frac{\max + \min}{2}$$

1. Determine whether the term periodic can be used to describe the graph for each situation. If so, state the period, equation of the axis and amplitude.

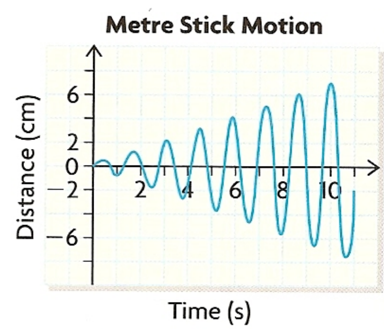
(a) the average number of hours of daylight over a 3-year period



(b) the motion of a piston on an automated assembly line



(c) a student is moving a metre stick back and forth with progressively larger movements



Assigned Work: WS 5-1; Textbook p. 235 #1-6, #8

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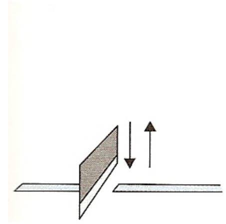
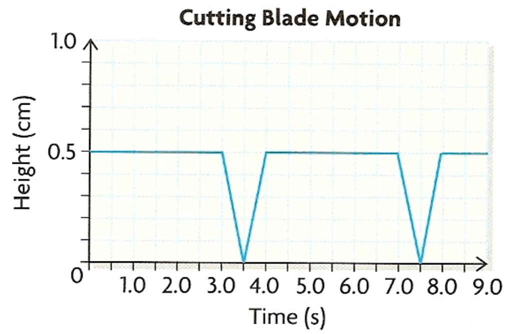
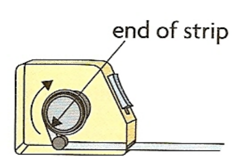
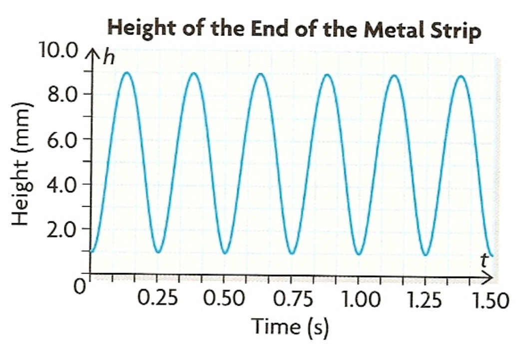
2. Tanya's mother works in a factory that produces tape measures. One day, Tanya and her brother Norman accompany their mother to work. During manufacturing, a metal strip is cut into 6 m lengths and is coiled within the tape measure holder. A cutting machine chops the strips into their appropriate lengths. Tanya's mother shows a graph that models the motion of the cutting blade on the machine in terms of time. How can Norman interpret the graph and relate its characteristics to the manufacturing process?





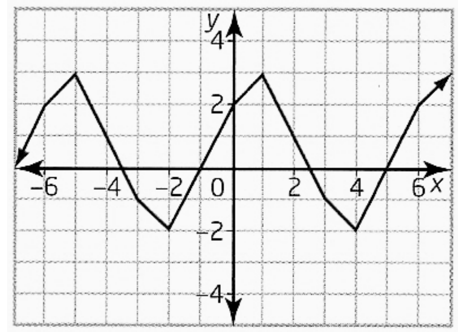
3. Further down the assembly line, the metal strip is raised and spooled onto a rotating cylinder contained within the tape measure. Tanya notices that the height of the end of the metal strip that attaches to the spool goes up and down as the rest of the strip is pulled onto the cylinder. Tanya's mother shows them the graph that models the height of the end of the strip in terms of time. How can Tanya interpret the graph and relate its characteristics to the manufacturing process?





4. For a periodic function $f(x)$ with period p, $f(x + p) = f(x)$, and $f(x + np) = f(x)$, where n is any integer. Consider the periodic function shown.

(a) What is the period of the function?



(b) Determine $f(2)$ and $f(5)$.

(c) Predict $f(8)$ and $f(-10)$.

(d) What is the amplitude of the function?

(e) Determine four x-values such that $f(x) = 2$.

Answers: 1. (a) periodic, 365 days, y = 12, 6 hours, (b) periodic, 6 sec, y = −2.5, 3.5 cm, (c) non-periodic**;**
4. (a) 6, (b) $f(2)=1$, $f(5)=0$, (c) $f(8)=1$, $f(-10)=1$, (d) 2.5, (e) $x=-12,-6,0,6$ **;**

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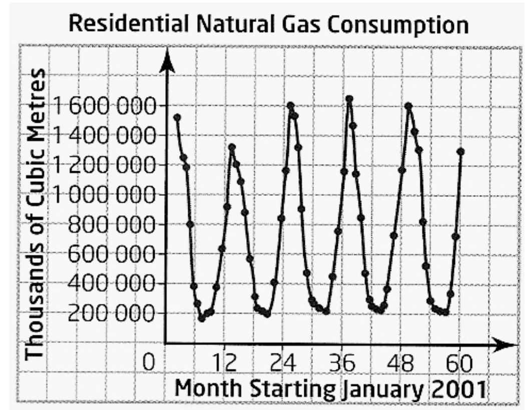
5. The graph shows residential natural gas consumption in Ontario per month, beginning in January 2001.



(a) Explain why the graph has this shape.

(b) Do the data appear to be periodic? Justify your answer.

(c) Assume that the consumption of natural gas in Ontario can be modelled using a periodic function. Determine the approximate maximum value, minimum value, and amplitude of this function.

(d) Estimate the period of this function. Does this value make sense? Explain why.

(e) Estimate the domain and range of the function.

(f) Explain how the graph can be used to estimate the natural gas consumption in February 2011.

Answers: 5. (a) Natural gas consumption is the highest in January which is usually the coldest month of a year. Then the weather starts to get warmer and reaches July which is usually the hottest month of a year and use of natural gas is at its lowest. Then the weather gets colder and colder again until January. The cycle continues to repeat years after years., (b) Data do appear almost periodic with a repeating pattern at regular intervals, except that the maximum values are not constant,
(c) max = 1.6 billion m³, min = 0.2 billion m³, Amp = 0.7 billion m³, (d) 12 months, it's the yearly weather cycle in Ontario, (e) $D = \{x \mid x \in R, 1 \le x \le 60\}$, $R = \{y \mid y \in R, 0.2 \text{ billion} \le y \le 1.6 \text{ billion}\}$
(f) Assume the graph is periodic and use the average natural gas consumption for February to estimate