The Fundamental Counting Principle
If you have (a) ways of doing event 1, (b) ways of doing event 2, and (c) ways of doing event 3, then you can find the total number of outcomes by multiplying.

$$a \times b \times c$$

Ex.1 A computer store sells 2 different types of computers, 3 different monitors, and 2 different printers. How many different computer systems are available?

$$2 \cdot 2 \cdot 3 = \boxed{12}$$

Factorial Notation!

$2! = 2 \cdot 1 = 2$
$3! = 3 \cdot 2 \cdot 1 = 6$
$4! = 4 \cdot 3 \cdot 2 \cdot 1 = 24$
$5! = 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1 = 120$
$6! = 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1 = 720$

Ex.2 simplify $\dfrac{7!}{5!}$

$$\frac{7 \cdot 6 \cdot 5!}{5!}$$

$$\frac{7!}{5!} = \frac{7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1}{5 \cdot 4 \cdot 3 \cdot 2 \cdot 1} = 42$$

Permutations
• An ordered arrangement of objects.

$$\boxed{nPr} = \frac{n!}{(n-r)!}$$

$n = \#$ of different objects
$r = \#$ chosen

Ex.3 How many three letter permutations can be formed using the letters from COMPUTE.

$$7 \; nPr \; 3 = \boxed{210}$$

$n = 7$

$r = 3$

Ex.4 There are 4 songs on your iPod and you shuffle the songs. How many permutations of all the songs are there?

$$4 \; nPr \; 4 = \boxed{24}$$

$n = 4$

$r = 4$

Combinations
- An unordered arrangement of objects.
- The number of (n) distinct objects taken (r) at a time given by

$$nCr = \frac{n!}{(n-r)! \, r!}$$

Ex.5 How many different groups of three people can be formed from seven people?

$$7 \; nCr \; 3 = \boxed{35}$$

$n = 7$

$r = 3$

Ex.6 If a teacher has enough money to buy four students a hamburger out of a class of ten. How many possible groups of students are there?

$$10 \; nCr \; 4 = \boxed{210}$$

$n = 10$

$r = 4$

Finding a Probability Using Permutations

Ex.1 A research laboratory requires a four digit security code to gain access to the facility. A security code can contain any digits 0-9 but no digit is repeated. What is the probability that a scientist is randomly assigned a code with the digits 1, 2, 3, and 4 in any order?

$$10 \, P \, 4 = 5040$$
$$4 \, P \, 4 = \boxed{24}$$

$$\frac{24}{5040} = \boxed{\frac{1}{210}}$$

Ex.2 A certain motorcycle license plate consists of a 5 digits that are randomly selected. No digit is repeated. What is the probability of getting a license plate consisting of all even digits?

$$10 \, P \, 5 = 30,240$$
$$5 \, P \, 5 = 120$$

$$\frac{120}{30,240} = \boxed{\frac{1}{252}}$$

There are 8 finalists in the 100 meter dash at the Olympic Games. Suppose 3 of the finalists are from the US.

Ex.3 What is the probability that the United States will win all 3 medals in this event.

$$8 \, C \, 3 = 56$$
$$3 \, C \, 3 = 1$$

$$\boxed{\frac{1}{56}}$$

Ex.4 What is the probability that the United States will win no medals in this event.

$$\boxed{\frac{5}{28}}$$

Finding the Number of Permutations with Repetition
Permutations with Permutations
The number of different permutations of n objects where one object repeats (a) times, a second object repeats (b) times, and so on is...
$$\frac{n!}{a! \times b! \times ...}$$

Ex.5 How many different permutations are there of the letters in the word ARKANSAS.

$$\frac{8!}{3! \, 2!} = \boxed{3,360}$$

Finding a Probability Using Permutations with Repetition
Ex.6 The school jazz band has 4 boys and 4 girls, and they are randomly line up for a yearbook photo.

a) Find the probability of getting an alternating boy girl pattern?

$$\frac{2}{70} \quad \boxed{\frac{1}{35}}$$

$$\frac{8!}{4! \, 4!} = 70$$

BGBGBGBG
GBGBGPGB
2

b) Find the probability of getting all of the boys together?

$$\frac{5}{70} = \boxed{\frac{1}{14}}$$

$$\frac{8!}{4! \, 4!} = 70$$

BBPBGGGG   GGGBBBPG
GBBBBBGGG   GGGGBBBU
GGBBBBGG   5

Ex.7 There are 2 mystery books, 2 romance books, and 2 poetry books to be randomly placed on a shelf. What is the probability that the mystery books are next to each other, the romance books are next to each other, and the poetry books are next to each other?

$$\frac{6!}{2! \, 2! \, 2!} = 90$$

$$\frac{6}{90} = \boxed{\frac{1}{15}}$$

MMRRPP   RRMMPe
MMPPRR   RRPeMM
PPRRMM   6
PeMMRR

Ex.8 what is the probability that a random arrangement of the letters in the word APPLE will have the two P's next to each other?

PP...   ALE   AEL

$$\frac{5!}{2!} = 60$$

$$\frac{24}{60} = \boxed{\frac{2}{5}}$$

$$\cdots PP \cdots$$
$$\cdots PP \cdot$$
$$\cdots PP$$

4

LEA LAE
EAL ELA

6

Finding a Probability Using Combinations

Ex.1 There are 4 boys and 8 girls on the debate team. The coach randomly chooses 3 of the students to participate in a competition. What is the probability that the coach chooses all girls?

$$_{12}C_3 = 220$$

$$_8C_3 = 56$$

$$\frac{56}{220} = \boxed{\frac{14}{55}}$$

Ex.2 There are 52 cards in a standard deck, 13 in each of 4 suits: clubs, diamonds, hearts, and spades. Five cards are drawn from the deck. What is the probability that all five cards are diamonds?

$$_{52}C_5 = 2,598,960$$

$$_{13}C_5 = 1287$$

$$\boxed{\frac{1287}{2,598,960}}$$

Ex.3 A coin is tossed 4 times, what is the probability of getting 3 heads?

$$2 \cdot 2 \cdot 2 \cdot 2 = 2^4 = 16$$

$$_4C_3 = 4$$

$$\frac{4}{16} = \boxed{\frac{1}{4}}$$

Ex.4 A standard deck of cards is divided in half, with the red cards separated from the black cards. Four cards are randomly drawn from the red half. What is the probability they are all diamonds?

$$_{26}C_4 = 14,950$$

$$_{13}C_4 = 715$$

$$\frac{715}{14,950} = \boxed{\frac{11}{230}}$$

Finding probability Using Combinations and Addition

- **Sometimes, counting problems involve the phrase "at least" or "at most". For these problem, Combinations must be added.**

Ex.5 A coin is flipped 5 times. What is the probability that the result is heads at least 4 of the 5 times?

$$2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 = 2^5 = 32 \qquad \frac{5+1}{32} = \boxed{\frac{3}{16}}$$

$$_5C_4 = 5$$
$$_5C_5 = 1$$

Ex.6 Three number cubes are rolled and the result is recorded. What is the probability that at least 2 of the number cubes show 6?

$$6 \cdot 6 \cdot 6 = 6^3 = 216 \qquad \frac{5(3)+1}{216} = \boxed{\frac{2}{27}}$$

$$_3C_2 = 5(3)$$
$$_3C_3 = 1$$

Ex.7 There are equally many boys and girls in the senior class. If 5 seniors are randomly selected to form the student council, what is the probability the council will contain at least 3 girls?

$$2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 = 2^5 = 32 \qquad \frac{10+5+1}{32} = \boxed{\frac{1}{2}}$$

$$_5C_3 = 10$$
$$_5C_4 = 5$$
$$_5C_5 = 1$$

Conditional Probability

The probability that event A occurs given that event B has already occurred is called the conditional probability of A given B and is written as P(A|B).

Finding conditional probability from a two way frequency table

	Took Medicine	No Medicine	Total
Headache	11	13	24
No Headache	54	22	76
Total	65	35	100

Ex.1 To the nearest percent, what is the probability that a participant who took the medicine did not get a headache?

$$\frac{54}{65} = .83 = 83\%$$

Ex.2 What is the probability that a participant who did not get a headache took medicine.
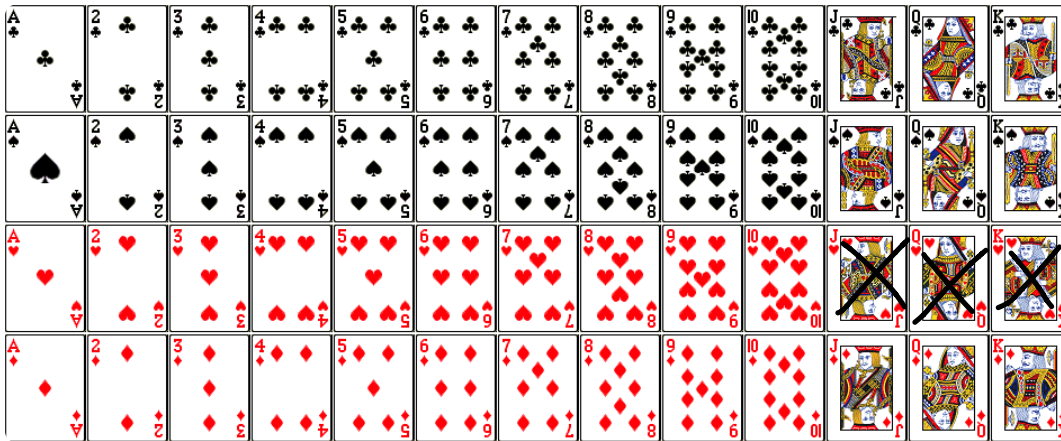
$$\frac{54}{76} = .71 = 71\%$$

Ex.3 What's the probability that a person has a headache, given they took medicine.

$$\frac{11}{65} = .17 = 17\%$$

Using the conditional probability formula
The conditional probability of A given B

$$P(A|B) = \frac{P(A \cap B)}{P(B)}$$

Ex.4 For a standard deck of playing cards, find the probability that a red card randomly drawn from the deck is a jack.
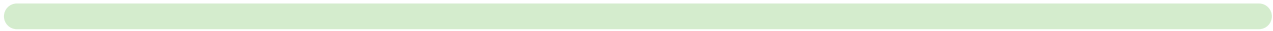
$$\frac{\frac{2}{52}}{\frac{26}{52}} = \frac{2}{26} = \boxed{\frac{1}{13}}$$

Ex.5 For a standard deck of playing cards, find the probability that a face card randomly drawn from the deck is a king.

$$\frac{4}{12} = \frac{1}{3}$$

Ex.6 For a standard deck of playing cards, find the probability that a queen randomly drawn from the deck is a diamond.

$$\boxed{\frac{1}{4}}$$

Expected Value

- **The expected value is the average of all possible outcomes, adjusted (or weighted) or the likelihood that each outcome will occur.**

$$E(x) = x_1 p_1 + x_2 p_2 + x_3 p_3 + \ldots + x_n p_n$$

$$P_n = \text{probability}$$

$$x_n = \text{winnings}$$

Ex.1 You draw one card from a standard deck of playing cards. If you pick a heart, you will win $10. If you pick a face card, which is not a heart, you win $8. If you pick any other card, you lose $6. Do you want to play?

$$\frac{13}{52}(10) + \frac{9}{52}(8) - \frac{30}{52}(6)$$

$$\frac{11}{26} = \boxed{.42}$$

Ex.2 The world famous gambler from Philadelphia, Señor Rick, proposes the following game of chance. You roll a fair die. If you roll 1, then Señor Rick pays you $25. If you roll a 2, Señor Rick pays you $5. If you roll a 3, you win nothing. If you roll a 4 or 5, you must pay Señor Rick $10, and if you roll a 6, you must pay Señor Rick $15. Is Señor Rick loco for proposing such a game? Explain.

$$①, ②, 3, 4, 5, 6$$

$$\frac{1}{6}(25) + \frac{1}{6}(5) + \frac{1}{6}(0) - \frac{2}{6}(10) - \frac{1}{6}(15)$$

$$-\frac{5}{6} = -.83 \qquad \boxed{No}$$

Ex.3 You pay $10 to play the following game of chance. There is a bag containing 12 balls, five are red, three are green and the rest are yellow. You are to draw one ball from the bag. You will win $14 if you draw a red ball and you will win $12 if you draw a yellow ball. How much do you expect to win or lose if you play the game 100 times?

5 red
3 green
4 yellow

$$\frac{5}{12}(4) + \frac{4}{12}(2) - \frac{3}{12}(10)$$

$$\left(-\frac{1}{6}\right) 100 = \boxed{\$-16.67}$$

Ex.4 A detective figures that he has a one in nine chance of recovering stolen property. His out of pocket expenses for the investigation are $9,000. If he is only paid if he recovers the stolen property, what should he charge clients in order to break even?

$$9(9000) = \left(\frac{1}{9}X\right)9$$

$$81,000 = X$$