Right Triangle Trig

- Trigonometry- the study of triangles
- Pythagorean Theorem, only works on right triangles. $a^2+b^2=c^2$

Trig Ratios

- Adjacent side- the leg next to an acute angle in a right triangle that is not the hypotenuse.
- Opposite side- the side across from an angle in a triangle.
- Hypotenuse- the side opposite the 90 degree angle in a right triangle.

SOH-CAH-TOA

Sine- $\frac{\text{opposite side}}{\text{hypotenuse}}$ $\sin\theta = \frac{opp}{hyp}$

Cosine- $\frac{\text{adjacent side}}{\text{hypotenuse}}$ $\cos\theta = \frac{adj}{hyp}$

Tangent- $\frac{\text{opposite side}}{\text{adjacent side}}$ $\tan\theta = \frac{opp}{adj}$

Reciprocal Trig Ratios

Cosecant - $\frac{\text{hypotenuse}}{\text{opposite side}}$

$$\csc\theta = \frac{hyp}{opp}$$

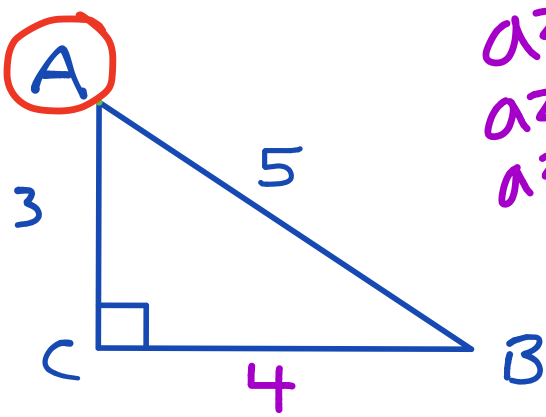
Secant - $\frac{\text{hypotenuse}}{\text{adjacent side}}$

$$\sec\theta = \frac{hyp}{adj}$$

Cotangent- $\frac{\text{adjacent side}}{\text{opposite side}}$

$$\cot\theta = \frac{adj}{opp}$$

Ex.1 Find all the trig ratios for angle A.



$$a^2+b^2=c^2$$
$$a^2+3^2=5^2$$
$$a^2+\cancel{9}=25$$
$$\cancel{-9} \quad -9$$
$$\sqrt{a^2}=\sqrt{16}$$
$$a=4$$

$\sin A = \frac{4}{5}$

$\cos A = \frac{3}{5}$

$\tan A = \frac{4}{3}$

$\csc A = \frac{5}{4}$

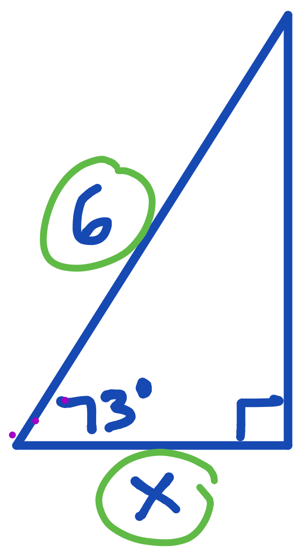
$\sec A = \frac{5}{3}$

$\cot A = \frac{3}{4}$

Solving for missing sides

- Choose the trig ratio that matches the given information, then solve for the missing side.

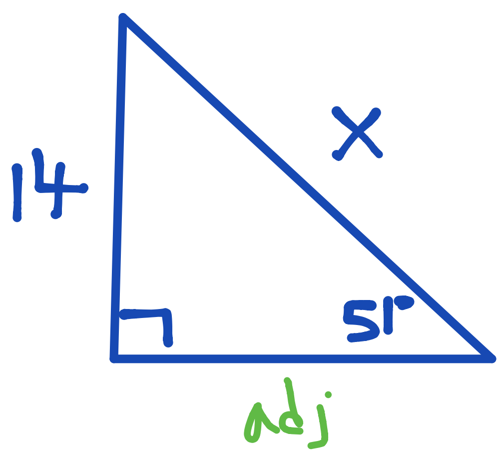
Ex.2 solve for x.



$$6\left(\cos 73^\circ\right) = \left(\frac{x}{6}\right)\cancel{6}$$

$$\boxed{x = 1.75}$$

Ex.3



$$x\left(\sin 51^\circ\right) = \left(\frac{14}{x}\right)\cancel{x}$$

$$\frac{x \cdot \cancel{\sin 51^\circ}}{\cancel{\sin 51}} = \frac{14}{\sin 51^\circ}$$

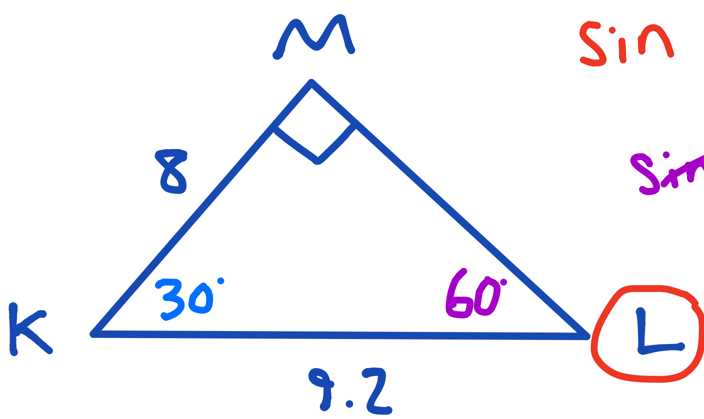
$$x = \frac{14}{\sin 51^\circ}$$

$$\boxed{x = 18}$$

Solving for the missing angle

- Choose the trig ratio that matches the given information, then solve for the missing angle by using arcsin, arccos, or arctan.

Ex.4 solve for the angles.



$\sin L = \frac{8}{9.2}$

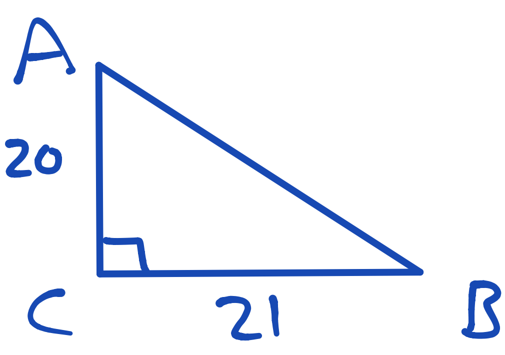
$\cancel{\sin^{-1}}\ \cancel{\sin} L = \sin^{-1}\left(\frac{8}{9.2}\right)$

$L = \sin^{-1}\left(\frac{8}{9.2}\right)$

$\boxed{L = 60^\circ}$

$180 - 90^\circ - 60^\circ$

Ex.5 Solve for angle B.

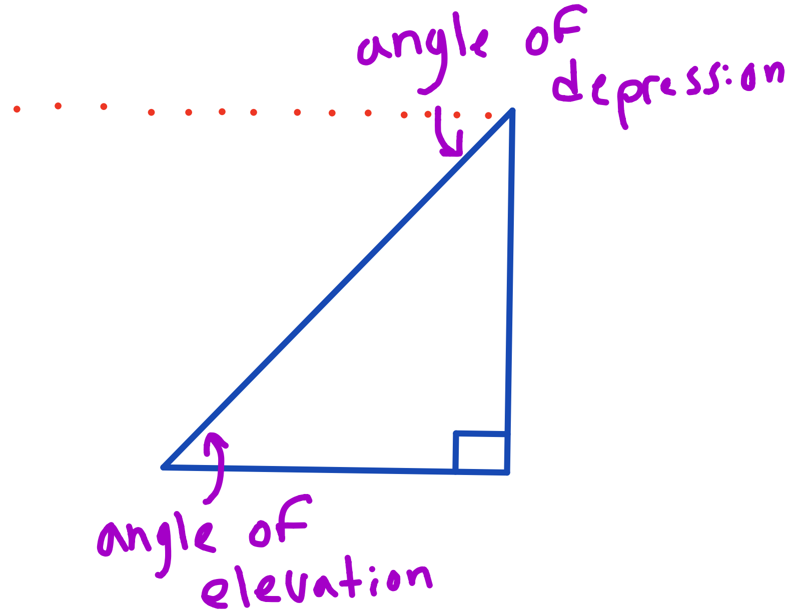


$\tan B = \frac{20}{21}$

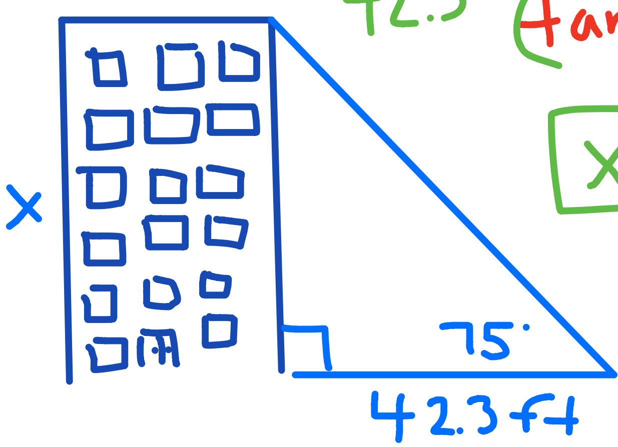
$B = \tan^{-1}\left(\frac{20}{21}\right)$

$\boxed{B = 44^\circ}$

Trig Application



Ex.5 At a point 42.3 feet from the base of a building, the angle of elevation of the top is 75 degrees. How tall is the building?



$$42.3\,(\tan 75) = \left(\frac{x}{\cancel{42.3}}\right)\cancel{42.3}$$

$$\boxed{x = 158\text{ ft}}$$

Law of Cosines

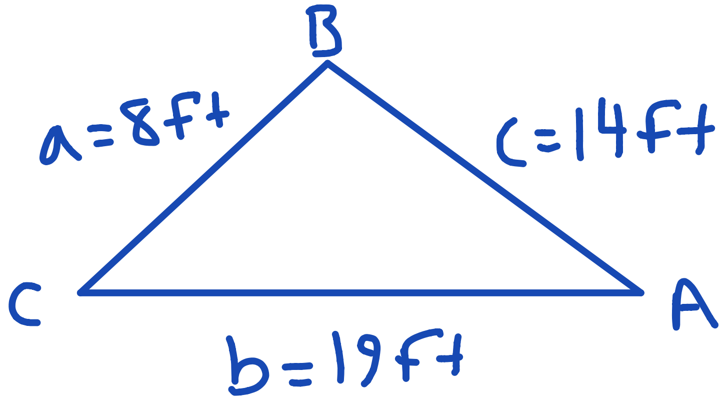
- If you are given three sides (SSS) or two sides and their included angle (SAS), Law of Cosines can be used.

$$a^2 = b^2 + c^2 - 2bc\cos A$$

$$b^2 = a^2 + c^2 - 2ac\cos B$$

$$c^2 = a^2 + b^2 - 2ab\cdot\cos C$$

Ex.1 (SSS) Find the three angles of the triangle.



$$a^2 = b^2 + c^2 - 2bc\cdot\cos A$$

$$8^2 = 19^2 + 14^2 - 2(19)(14)\cdot\cos A$$

$$64 = \cancel{557} - 532\boxed{\cos A}$$

$$-557 \quad -557$$

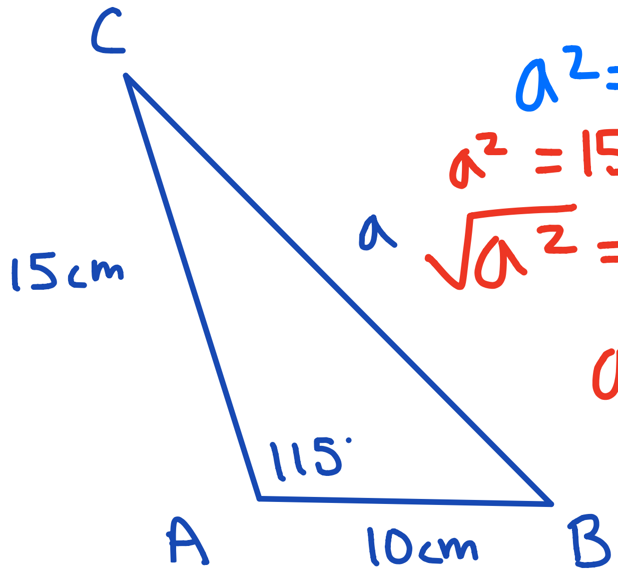
$$\frac{-493}{-532} = \frac{\cancel{-532}\cos A}{\cancel{-532}}$$

$$.93 = \cos A$$

$$\cos^{-1}(.93) = A$$

$$\boxed{A = 22^\circ}$$

Ex.2 (SAS) Find the remaining two angles and the side of the triangle.



$$a^2 = b^2 + c^2 - 2bc\cos A$$

$$a^2 = 15^2 + 10^2 - 2(15)(10)\cos 115°$$

$$\sqrt{a^2} = \sqrt{451.79}$$

$$a = \sqrt{ans}$$

$$\boxed{a = 21.3}$$

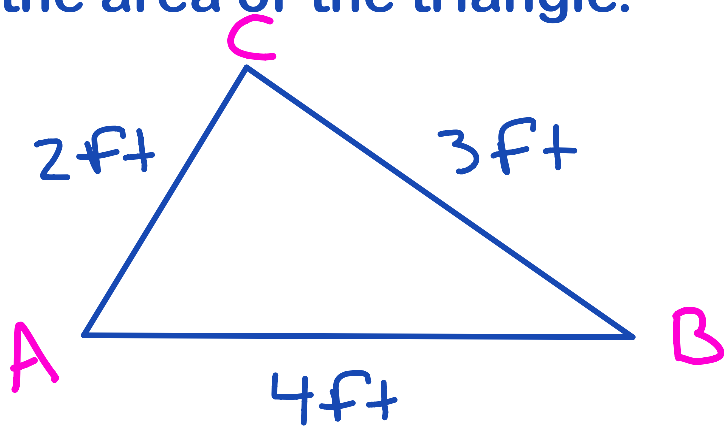
Heron's Area Formula

- The Law of Cosines can be used to establish the following formula.

$$A = \sqrt{s(s-a)(s-b)(s-c)}$$

where $s = (a+b+c)/2$

Ex.3 Find the area of the triangle.



$$A = \sqrt{s(s-a)(s-b)(s-c)}$$

where $s = (a+b+c)/2$ $\quad s = 4.5$

$$A = \sqrt{4.5(4.5-3)(4.5-2)(4.5-4)}$$

$$\boxed{A = 2.9}$$

$$a^2 = \cancel{b^2} + \cancel{c^2} - 2bc\cos A$$

$$-b^2 - c^2 \quad \cancel{-b^2} \quad -c^2$$

$$\frac{a^2 - b^2 - c^2}{-2bc} = \frac{\cancel{-2bc} \cdot \cos A}{\cancel{-2bc}}$$

$$\frac{a^2 - b^2 - c^2}{-2bc} = \cos A$$

$$\boxed{A = \cos^{-1}\left(\frac{a^2 - b^2 - c^2}{-2bc}\right)}$$

Law of Sines

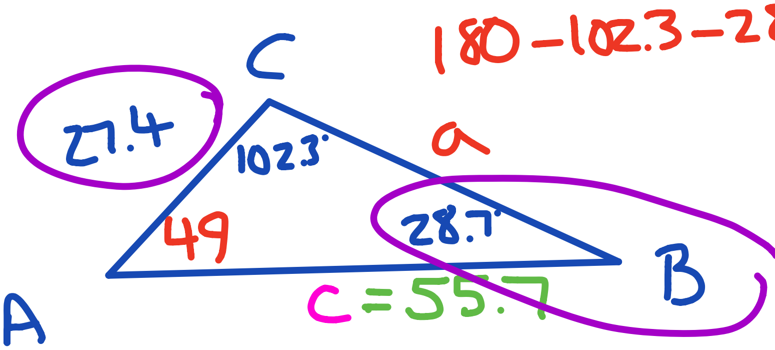
- **Oblique triangle- triangles that have no right angles**
- **If you are given two angles and any sides (AAS or ASA), or two sides and an angle opposite one of them (SSA), you can use The Law of Sines.**

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$$

Ex.1 AAS Find the remaining angles and sides.

180 − 102.3 − 28.7



$$\sin 102.3\left(\frac{27.4}{\sin 28.7}\right) = \left(\frac{c}{\cancel{\sin 102.3}}\right)\cancel{\sin 102.3}$$

$$c = \frac{\sin 102.3\,(27.4)}{\sin 28.7}$$

$$c = 55.7$$

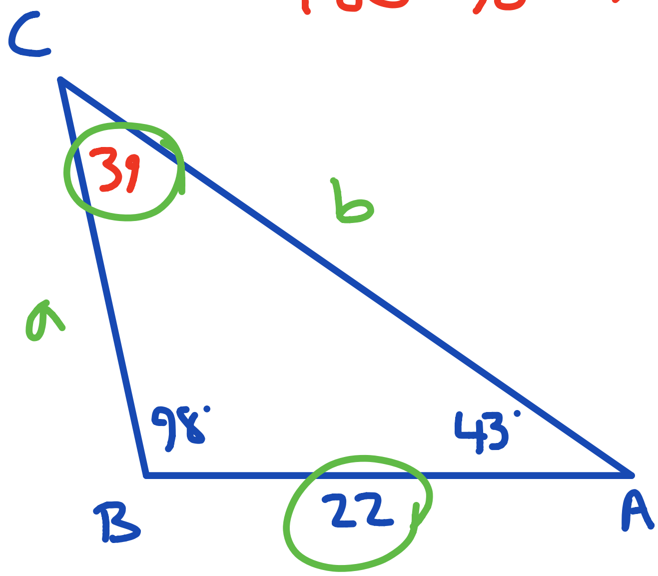
$$\frac{\sin 28.7}{27.4} = \frac{\sin 49}{a}$$

$$\sin 49\left(\frac{27.4}{\sin 28.7}\right) = \left(\frac{a}{\cancel{\sin 49}}\right)\cancel{\sin 49}$$

$$\boxed{a = 43}$$

Ex.2 ASA Find the remaining angles and sides.

$180 - 98 - 43$



$$\frac{22}{\sin 39} = \frac{b}{\sin 98}$$

$$\boxed{b = 35}$$

$$\frac{22}{\sin 39} = \frac{a}{\sin 43}$$

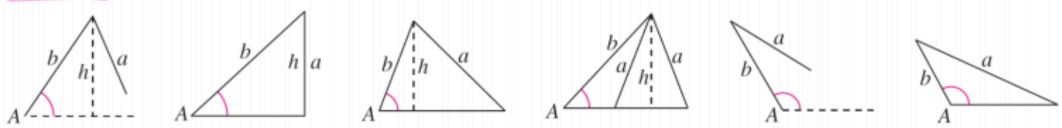
$$\boxed{a = 24}$$

The Ambiguous Case (SSA)

- **If two sides and one opposite angle are given, three possible situations can occur: no Triangle exists, one triangle exist, or two triangles exist.**

The Ambiguous Case (SSA)

Consider a triangle in which you are given a, b, and A. $(h = b \sin A)$

	A is acute.	A is acute.	A is acute.	A is acute.	A is obtuse.	A is obtuse.
Sketch						
Necessary condition	$a < h$	$a = h$	$a \geq b$	$h < a < b$	$a \leq b$	$a > b$
Triangles possible	None	One	One	Two	None	One

Ex.1 Determine the number of possible solutions for the triangle. A=30, a=8, b=10.

$h = b \sin A$

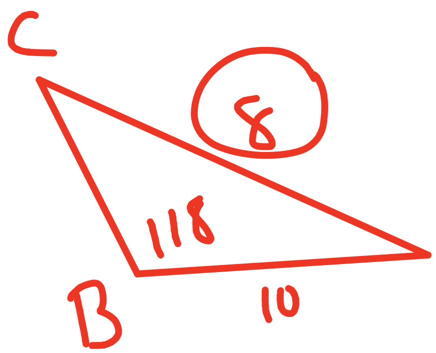
$h = 10(\sin 30)$

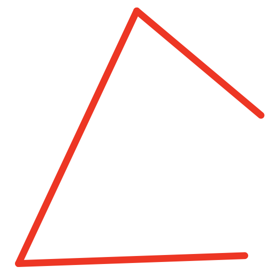
$h = 5$

$8 > 5$

$\boxed{\text{one}}$

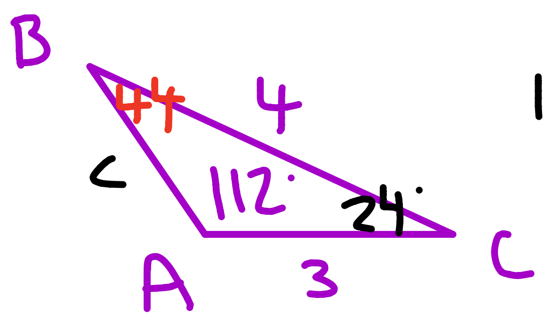
Ex.2 Determine the number of possible solutions for the triangle. b=8, c=10, B=118.





$$\boxed{\text{Zero } \Delta\text{'s}}$$

Ex.3 Find all the solutions for the triangle. a=4, b=3, A=112.



$$180-44-112=24$$

$$\frac{\sin A}{a}=\frac{\sin B}{b}$$

$$3\left(\frac{\sin 112}{4}\right)=\left(\frac{\sin B}{3}\right)\cancel{3}$$

$$\sin^{-1}\ .7=(\sin B)\ \sin^{-1}$$

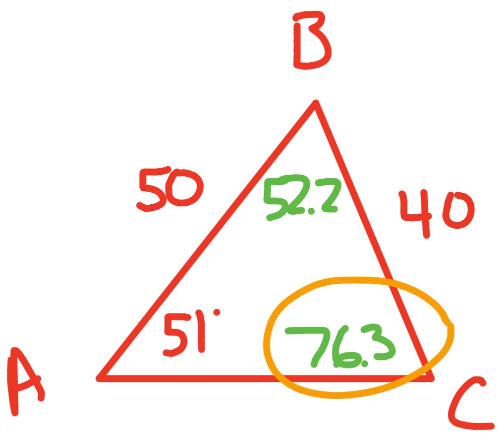
$$B=\underline{\sin^{-1}(.7)}$$

$$\cancel{\sin 24}\left(\frac{c}{\cancel{\sin 24}}\right)=\left(\frac{4}{\sin 112}\right)\sin 24$$

$$\boxed{c=1.8}$$

$\boxed{B = 44^\circ}$

Ex.4 Find all the solutions for the triangle.
A=51, a=40, c=50.



$$\frac{\sin C}{50} = \frac{\sin 51^\circ}{40}$$

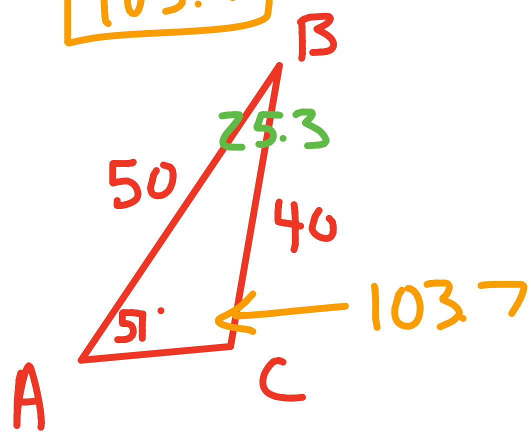
$$C = 76.3$$

$$\frac{b}{\sin(52.2)} = \frac{40}{\sin 51^\circ}$$

$$b = 40.9$$

$$180 - 76.3 = 103.7$$

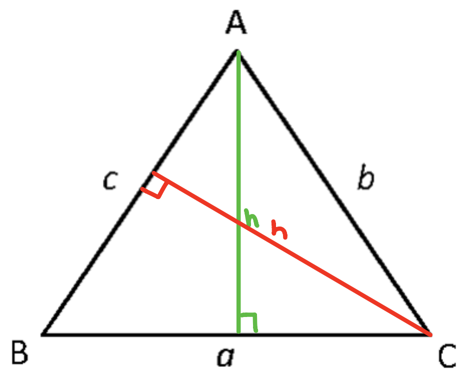
$$\boxed{103.7} + 51 = 154.7$$



$$\frac{b}{\sin(25.3)} = \frac{40}{\sin 51^\circ}$$

$$b = 22$$

Proving the Law of Sines



$c(\sin B) = \left(\frac{h}{\cancel{c}}\right)\cancel{c}$

$h = c(\sin B)$

$b \sin C = \left(\frac{h}{\cancel{b}}\right)\cancel{b}$

$h = b \sin C$

$\sin B = \frac{h}{a}$

$h = a \sin B$

$A = \frac{h}{b}$

$= b \sin A$

$\frac{\cancel{c}\sin B}{b\cancel{c}} = \frac{\cancel{b}\sin C}{\cancel{b}c}$

$\frac{\sin B}{b} = \frac{\sin C}{c}$

$\frac{\cancel{a}\sin B}{\cancel{a}b} = \frac{\cancel{b}\sin A}{a\cancel{b}}$

$\frac{\sin B}{b} = \frac{\sin A}{a}$

Transitive Property

$\frac{\sin B}{b} = \frac{\sin A}{a} = \frac{\sin C}{c}$

Ambiguous Case

- Occurs when you are given two consecutive sides and an angle. (SSA)
- 3 cases: no triangles, one triangle, two triangles.

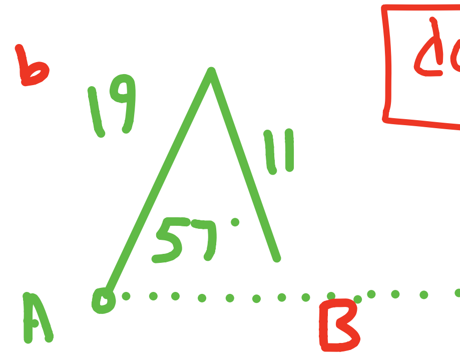
No triangles.

- When the given angle is obtuse the side opposite that angle must be the largest side.
- When the given angle is acute, the side opposite that angle must be greater than or equal to the altitude.
- Domain error in the calculator

1. $a = 19, b = 17, B = 93^o$

no △

2. $A = 57^o, a = 11, b = 19$



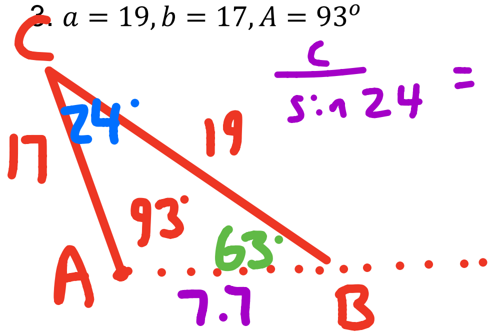
$\boxed{\text{domain error}}$

$\frac{\sin 57}{11} = \frac{\sin B}{19}$

One triangle.

- When the given angle is obtuse and the side opposite that angle is the longest side.
- When the given angle is acute and the side opposite that angle is equal to the length of the altitude. (right triangle)
- When the side opposite of the acute angle is longer than the altitude.

3. $a = 19, b = 17, A = 93^o$

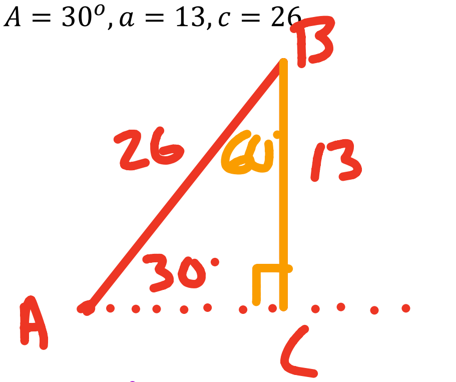


$$\frac{c}{\sin 24} = \frac{19}{\sin 93}$$

$$\frac{\sin B}{17} = \frac{\sin 93}{19}$$

$B = 63$

4. $A = 30^o, a = 13, c = 26$

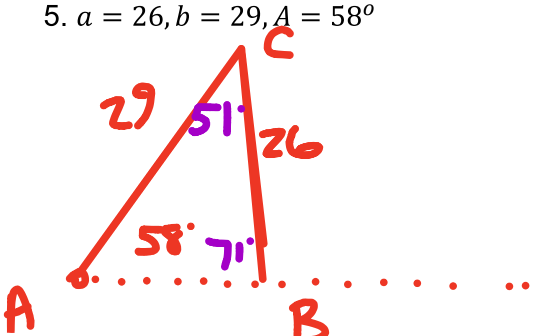


$$\frac{\sin C}{26} = \frac{\sin 30}{13}$$

Two Triangles

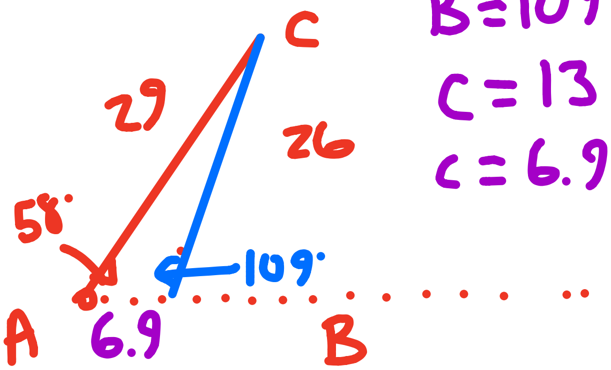
- When the given angle is acute the side opposite that angle is less than the other given side.

5. $a = 26, b = 29, A = 58^o$



$$\boxed{\begin{array}{l} B = 71 \\ C = 51^\circ \\ c = 23.8 \end{array}}$$

$$\frac{c}{\sin 51} = \frac{26}{\sin 58}$$

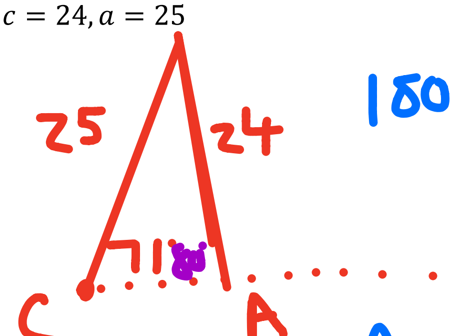


$B = 109$

$C = 13$

$c = 6.9$

6. $C = 71^o, c = 24, a = 25$



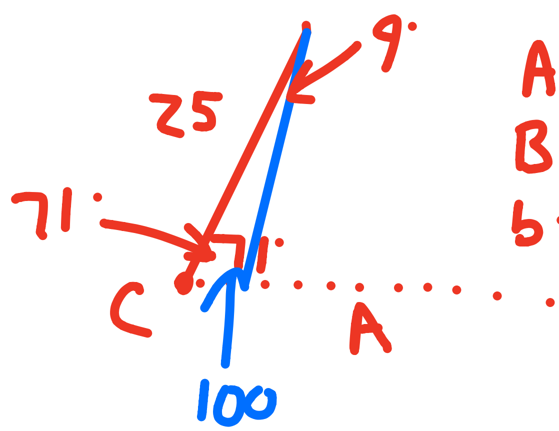
$180 - 80$

100

$A = 80^\circ$

$B = 29^\circ$

$b = 12.3$



$A = 100^\circ$

$B = 9^\circ$

$b = 3.9$