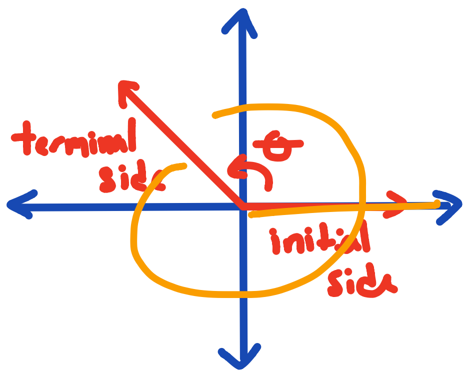
Angles and Degree Measures

- An angle with its vertex at the origin and its initial side along the positive x-axis is said to be in standard position.



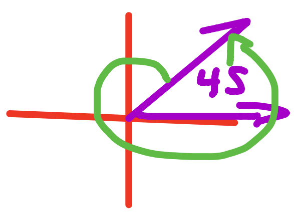
- Two angles in standard position are coterminal angles if they have the same terminal side. Since angles differing in degree measure by multiples of 360 are equivalent, every angle has infinitely many coterminal angles.

Ex.1 identify coterminal angles for...

A) 45°

$+360^\circ$

-360°



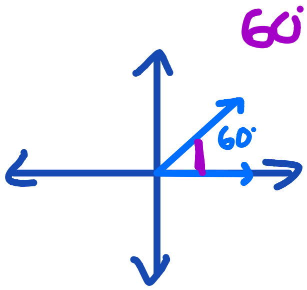
$\boxed{\begin{array}{c} 405^\circ \\ -315^\circ \end{array}}$

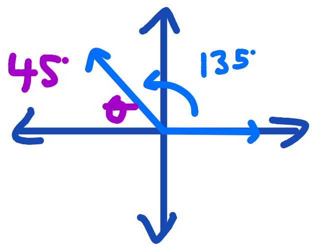
B) $225^\circ \pm 360^\circ$

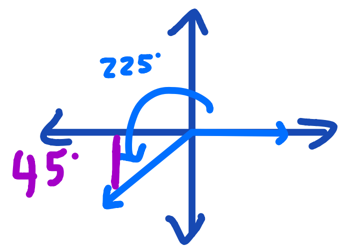
$\boxed{\begin{array}{c} 585^\circ \\ -135^\circ \end{array}}$

- **Reference angle- is defined as the acute angle formed by the terminal side and the x-axis.**

$180-135$ $\qquad$ $225-180$

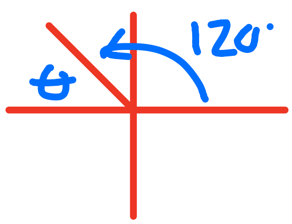






Ex.2 Find the measure of the reference angle for each angle.

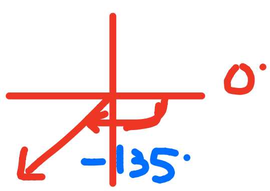
A) 120



$\boxed{60^\circ}$

B) -135

$180-135=45^\circ$



Radian- another type of angle measure. $\theta = \frac{s}{r}$

$1 \text{ radian} = \frac{180}{\pi} \text{ degrees} \approx 57.3^\circ$

$1 \text{ degree} = \frac{\pi}{180} \text{ radians or } 0.17 \text{ radians}$

Ex.3 Change 330° to radians.

$$\frac{330^{\circ}}{1}\cdot\frac{\pi}{180^{\circ}} = \frac{33\cancel{0}\pi}{18\cancel{0}} = \boxed{\frac{33\pi}{18}}$$

Ex.4 change $\frac{2\pi}{3}$ Radians to degree measure.

$$\frac{2\cancel{\pi}}{3}\cdot\frac{180}{\cancel{\pi}} = \frac{360}{3} = \boxed{120^{\circ}}$$

Ex.5 identify one positive and one negative coterminal angle for... $\pm 2\pi$

A) π $\boxed{3\pi, -\pi}$

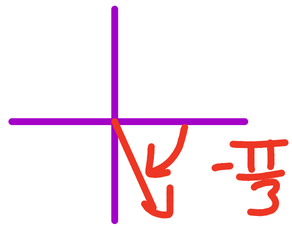
B) $\frac{\pi}{2}$ $\boxed{\frac{5\pi}{2}, -\frac{3\pi}{2}}$

C) $\frac{-7\pi}{3}$ $\boxed{-\frac{\pi}{3}, \frac{5\pi}{3}}$

$+2\pi$
$+2\pi$

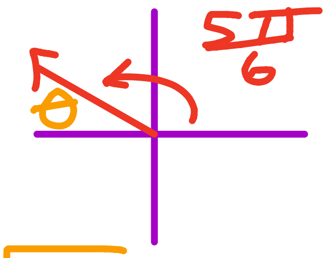
Ex.6 Sketch the angle then find the measure of the reference angle for each angle.

A) $-\frac{\pi}{3}$



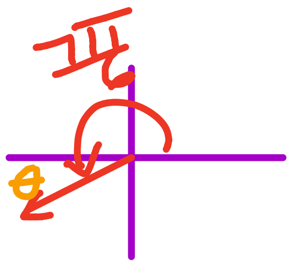
$\boxed{\frac{\pi}{3}}$

B) $\frac{5\pi}{6}$



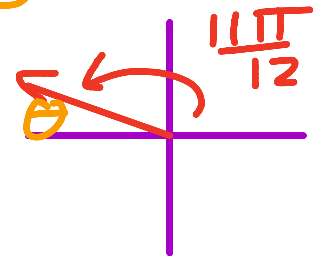
$\boxed{\frac{\pi}{6}}$

C) $\frac{7\pi}{6}$



$\boxed{\frac{\pi}{6}}$

D) $\frac{11\pi}{12}$

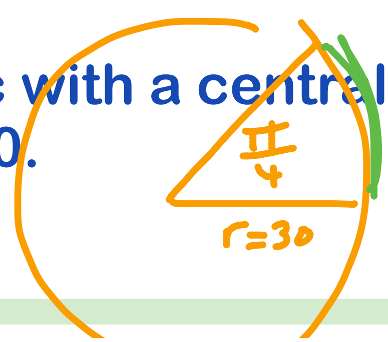


$\boxed{\frac{\pi}{12}}$

Arc Length Formula $s = r\theta$

- **s is the arc length**
- **r is the radius**
- **theta is the angle produced by the arc in radians.**

Ex.7 Find the length of an arc with a central angle of $\frac{\pi}{4}$ and a radius of 30.



$s = r\theta$

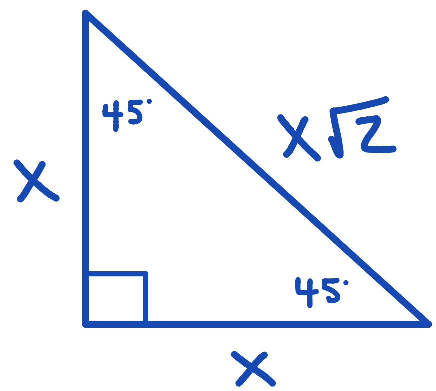
$s = 30\left(\frac{\pi}{4}\right)$

$s = \frac{15\pi}{2}$

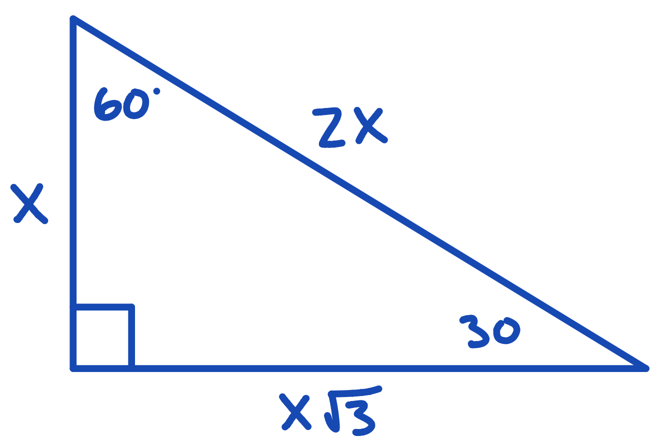
<u>Special Right Triangles</u>

- Right triangles whose angle measures are 45-45-90 or 30-60-90.

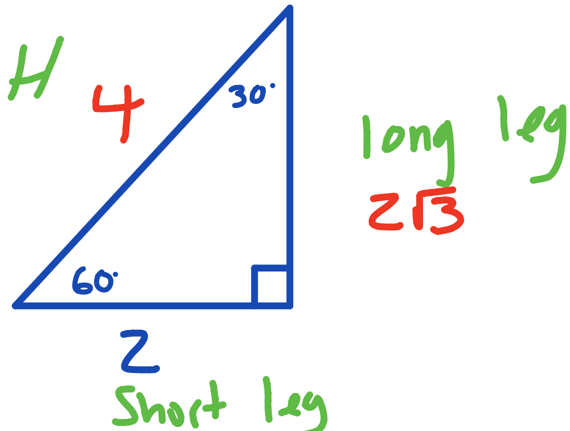
45-45-90: the hypotenuse is $\sqrt{2}$ times as long as the leg.



30-60-90: the hypotenuse is twice as long as the shorter leg, and the longer leg is $\sqrt{3}$ times as long as the shorter leg.

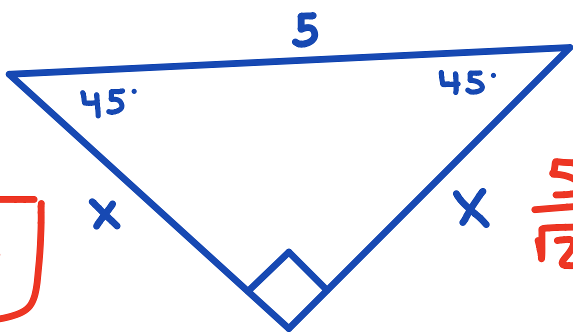


Ex.1 Find the missing sides



Ex.2 Find the value of x.

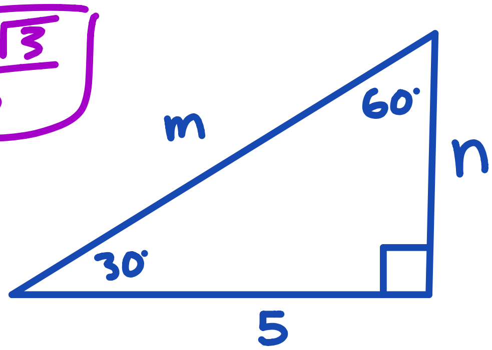
$\boxed{\frac{5\sqrt{2}}{2}}$



$$\frac{5}{\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}} = \frac{5\sqrt{2}}{\sqrt{4}} = \boxed{\frac{5\sqrt{2}}{2}}$$

Ex.3 Find the value of m and n.

$\boxed{\frac{10\sqrt{3}}{3}}$



$$\frac{5}{\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}} = \boxed{\frac{5\sqrt{3}}{3}}$$

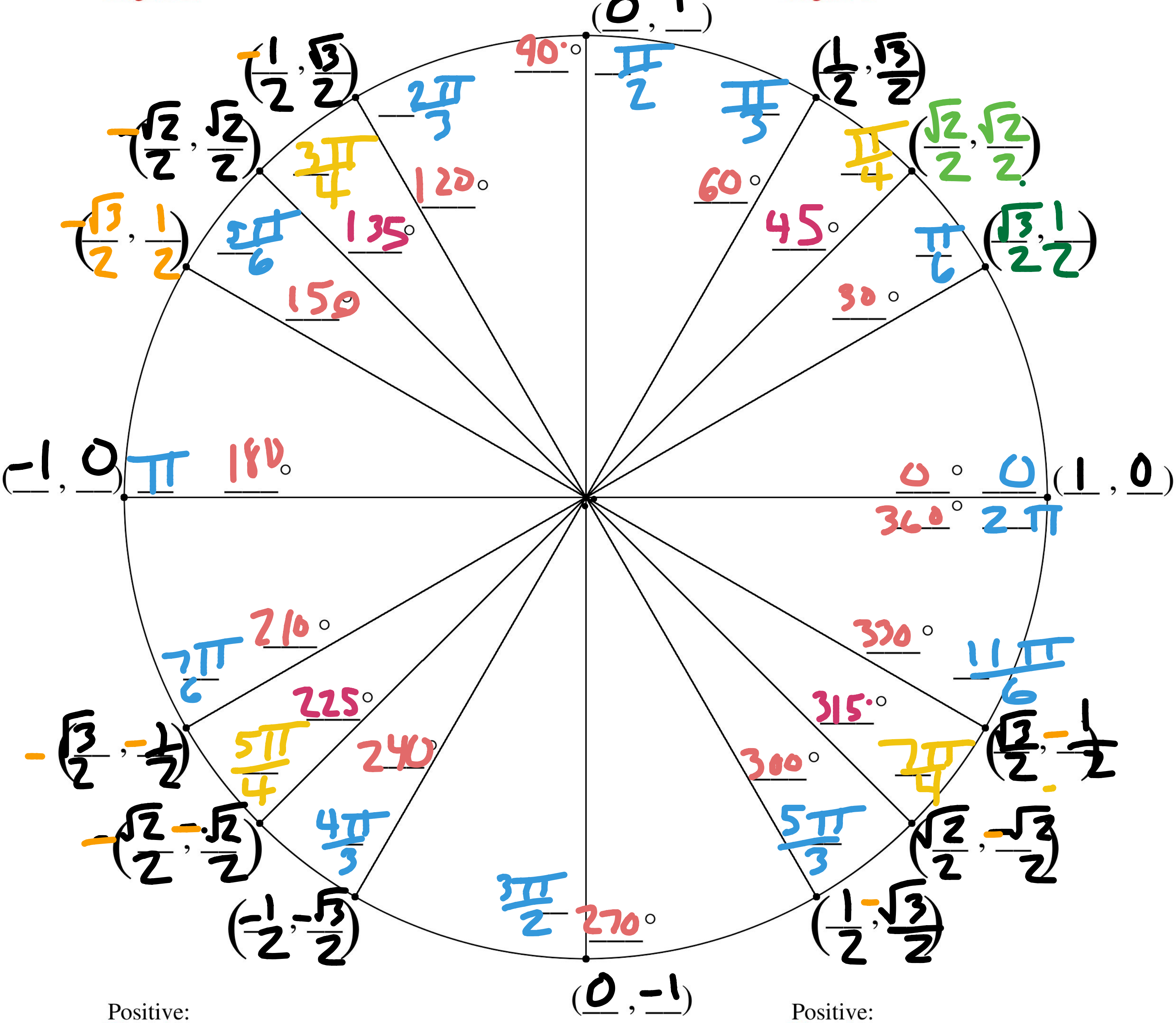
(cosine, sine)

Fill in The Unit Circle

(cosθ, sinθ)

Positive:
Negative:

Positive:
Negative:



Positive:
Negative:

Positive:
Negative:

$\tan\theta = \frac{y}{x}$

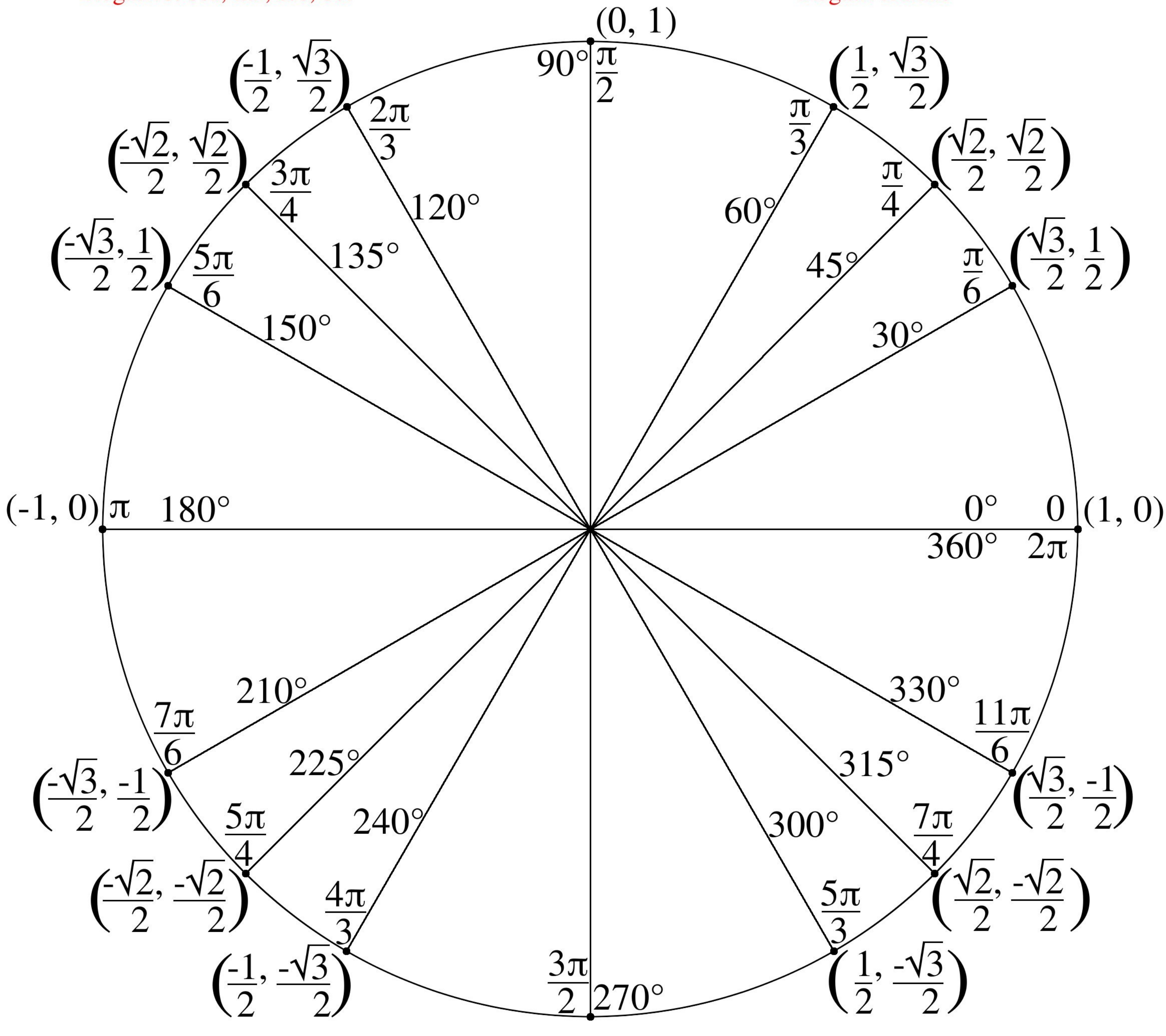
The Unit Circle

(x, y)

$(\cos\theta, \sin\theta)$

Positive: sin, csc
Negative: cos, tan, sec, cot

Positive: sin, cos, tan, sec, csc, cot
Negative: none



Positive: tan, cot
Negative: sin, cos, sec, csc

Positive: cos, sec
Negative: sin, tan, csc, cot

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Evaluating Trigonometric Ratios

$\tan\theta = \frac{\sin\theta}{\cos\theta}$

$\csc\theta = \frac{1}{\sin\theta}$

$\sec\theta = \frac{1}{\cos\theta}$

$\cot\theta = \frac{1}{\tan\theta}$

Ex.1 cos(150) $= \boxed{-\frac{\sqrt{3}}{2}}$

Ex.2 sin(-90) $= \boxed{-1}$

Ex.3 tan($\frac{-\pi}{6}$) $= \frac{-1}{\sqrt{3}} = \boxed{\frac{-\sqrt{3}}{3}}$

Ex.4 csc(225) $= \frac{-2}{\sqrt{2}} = \boxed{-\sqrt{2}}$

Ex.5 cos(3π) $= \boxed{-1}$

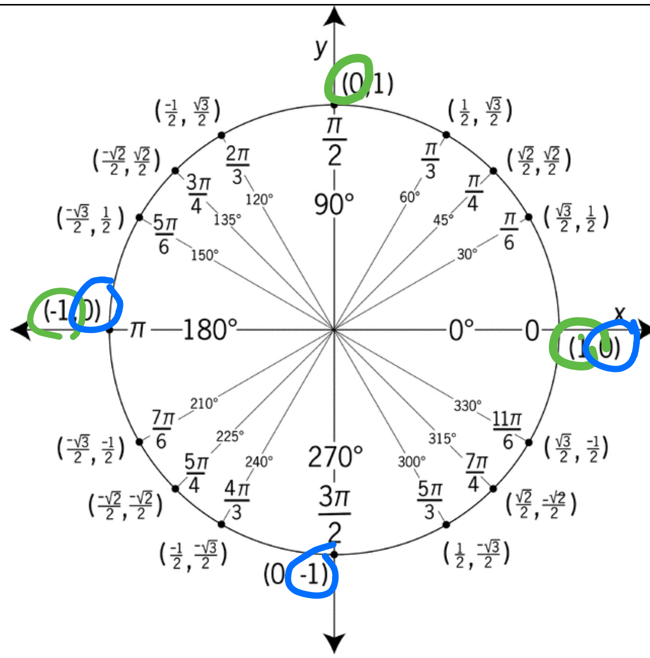
Ex.6 sec(945) $= \frac{-2}{\sqrt{2}} = \boxed{-\sqrt{2}}$

$f(\theta) = \cos\theta$

θ	$f(\theta)$
0	1
$\frac{\pi}{2}$	0
π	-1
$\frac{3\pi}{2}$	0
2π	1

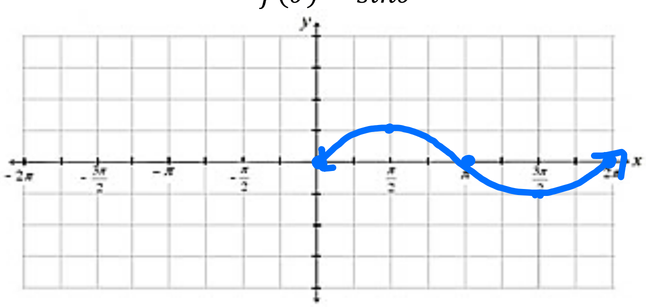
$f(\theta) = \sin\theta$

θ	$f(\theta)$
0	0
$\frac{\pi}{2}$	1
π	0
$\frac{3\pi}{2}$	-1
2π	0



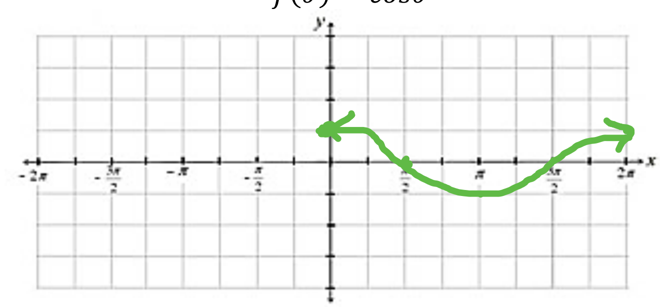
Parent Functions

$f(\theta) = sin\theta$

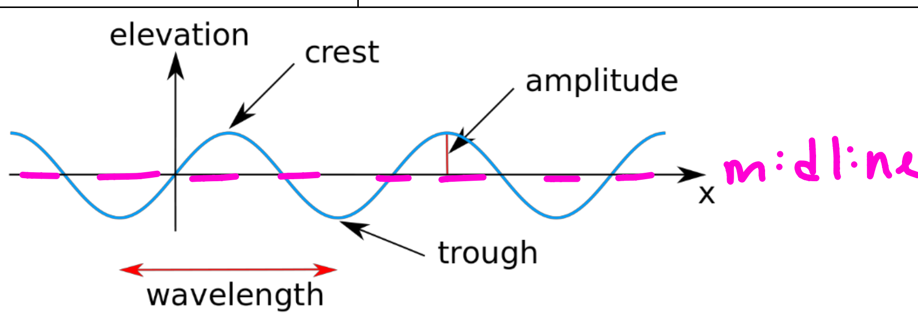


The sine function looks like a S.
Standard equation: $f(\theta) = a \cdot \sin\left(b(\theta - h)\right) + k$

$f(\theta) = cos\theta$



The cosine function looks like a cup
Standard equation: $f(\theta) = a \cdot \cos\left(b(\theta - h)\right) + k$



Amplitude (a) is the height from the center line to the peak (or to the trough). Or we can measure the height from highest to lowest points and divide that by 2.

Period the change in x to complete one full cycle. $Period = \frac{2\pi}{b}$

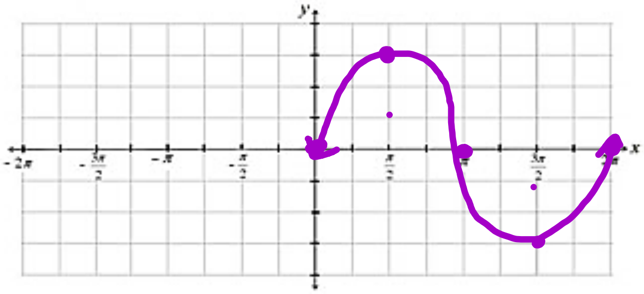
Vertical Shift (k) is how far the function is shifted **vertically** from the usual position.

Phase Shift (h) is how far the function is shifted **horizontally** from the usual position.

Amplitude

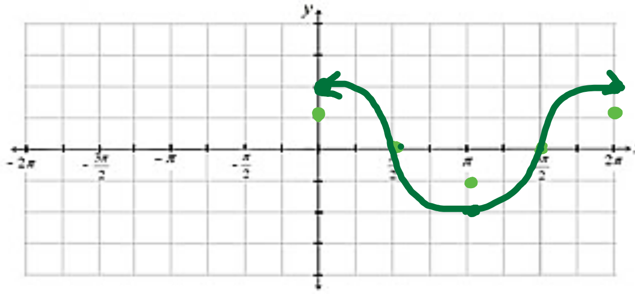
Ex. $f(\theta) = 3sin\theta$

Amplitude: 3 period: 2π



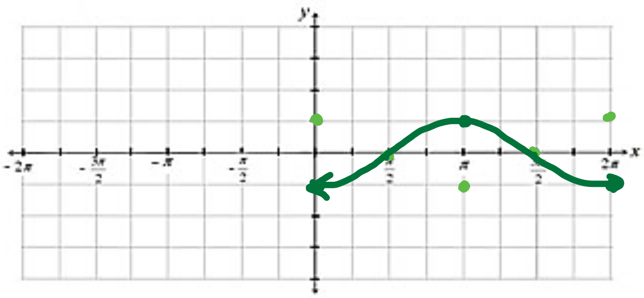
Ex. $f(\theta) = 2cos\theta$

Amplitude: 2 period: 2π



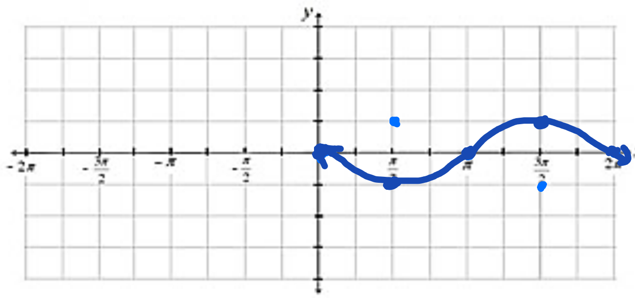
Ex. $f(\theta) = -cos\theta$

Amplitude: 1 period: 2π



Ex. $f(\theta) = -sin\theta$

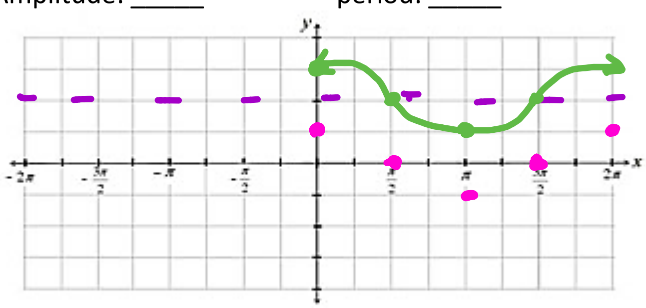
Amplitude: 1 period: 2π



Vertical Shifts

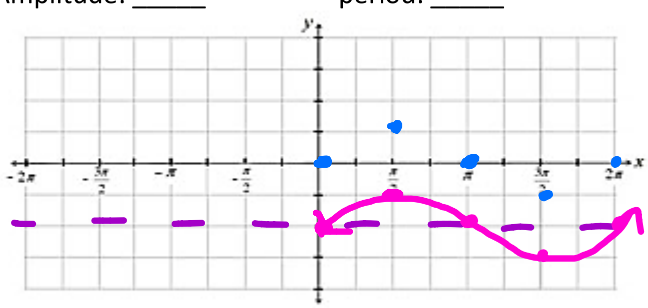
Ex. $f(\theta) = cos\theta + 2$

Amplitude: 1 period: 2π



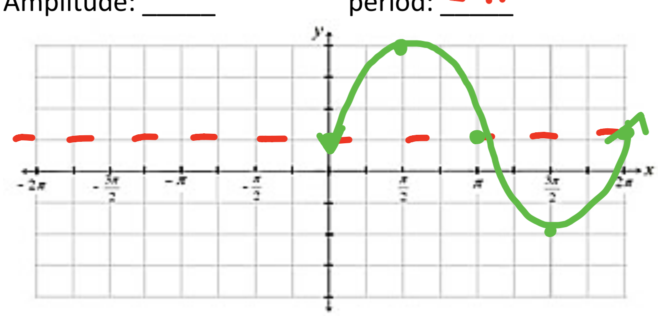
Ex. $f(\theta) = sin\theta - 2$

Amplitude: 1 period: 2π



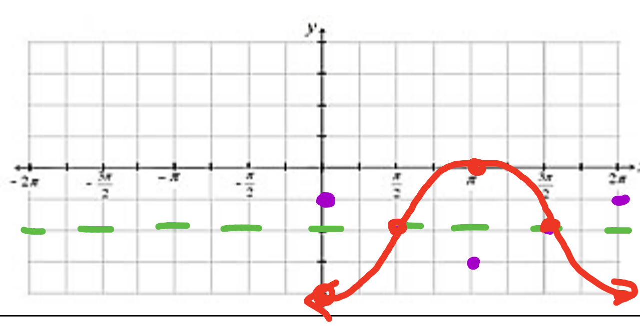
Ex. $f(\theta) = 3sin\theta + 1$

Amplitude: ______ period: 2π



Ex. $f(\theta) = -2cos\theta - 2$

Amplitude: 2 period: 2π



Use the given information to create a sine function

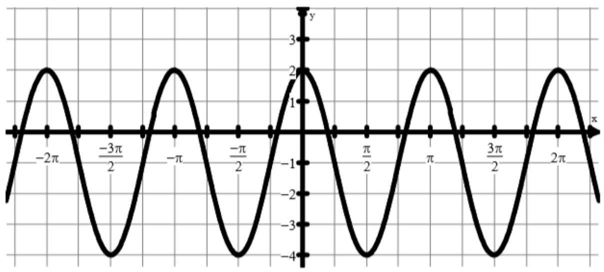
Ex.
Amplitude: 5
Period: 2π
Vertical Shift: down 4

$f(x) = 5\sin x - 4$

Ex.
Amplitude: 3
Period: 2π
Vertical Shift: up 1

$f(x) = 3\sin x + 1$

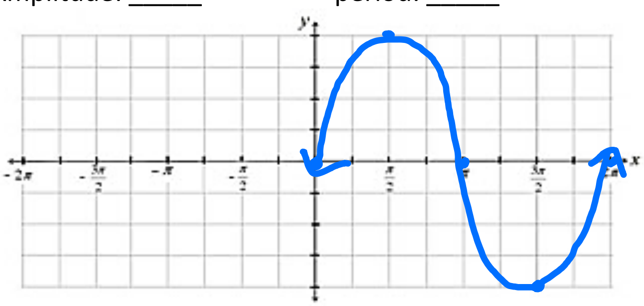
Ex. Write one cosine function for the graph.



Practice

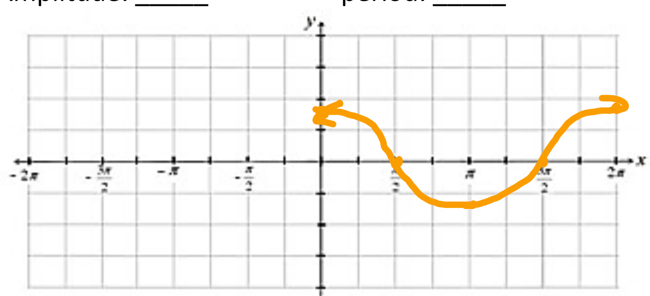
1. $f(\theta) = 4\sin\theta$

Amplitude: 4 period: 2π



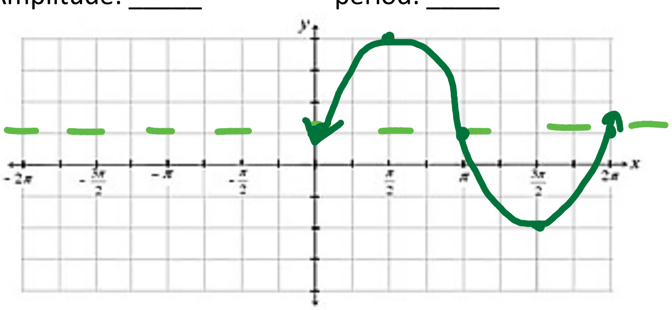
2. $f(\theta) = 1.5\cos\theta$

Amplitude: 1.5 period: 2π



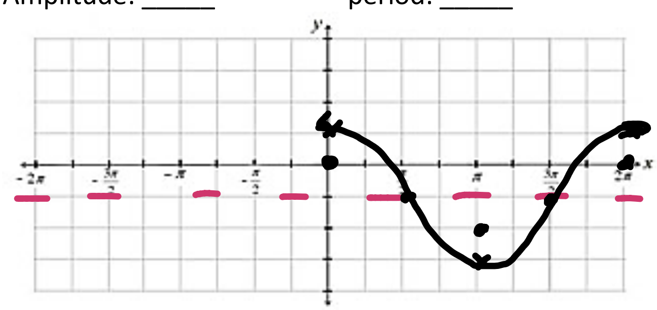
3. $f(\theta) = 3\sin\theta + 1$

Amplitude: 3 period: 2π



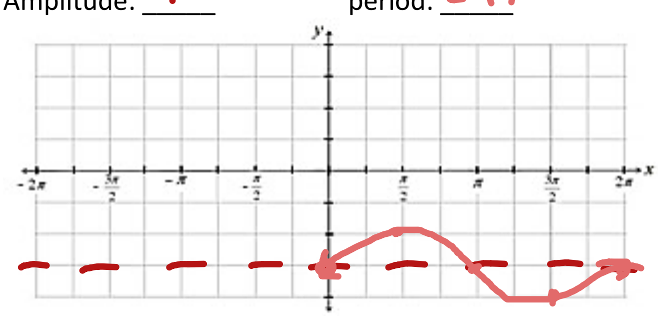
4. $f(\theta) = 2\cos\theta - 1$

Amplitude: 2 period: 2π



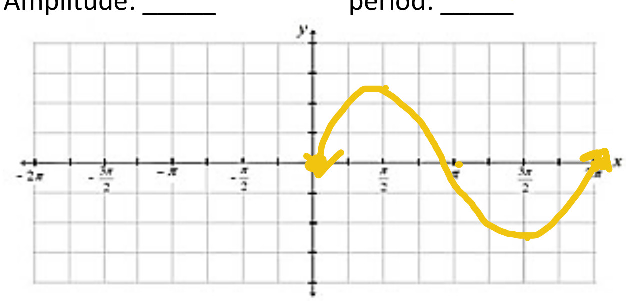
5. $f(\theta) = \sin\theta - 3$

Amplitude: 1 period: 2π



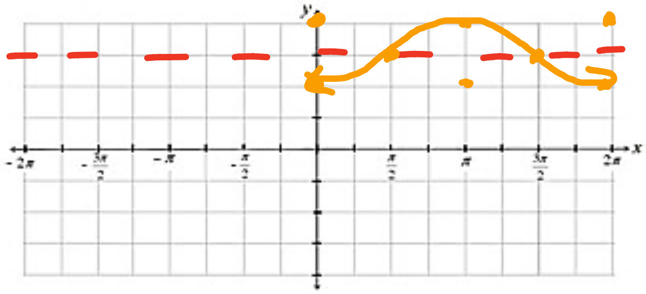
6. $f(\theta) = \frac{5}{2}\sin\theta$

Amplitude: 2.5 period: 2π



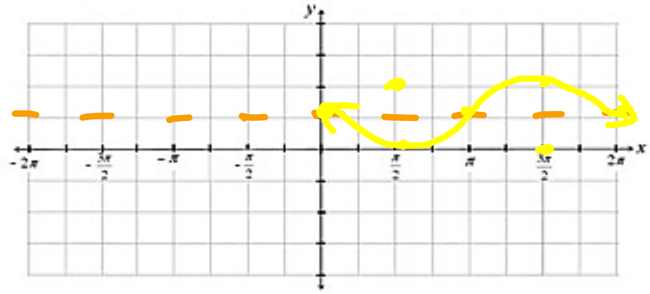
7. $f(\theta) = -cos\theta + 3$

Amplitude: 1　　　　period: 2π



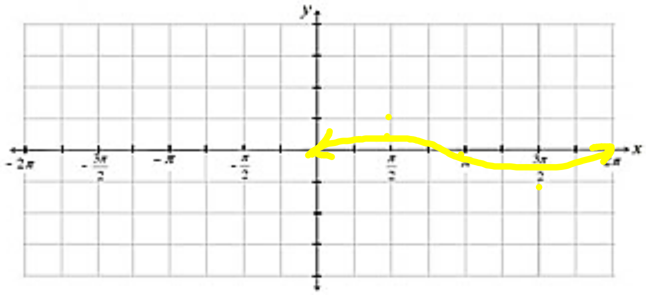
8. $f(\theta) = -sin\theta + 1$

Amplitude: 1　　　　period: 2π



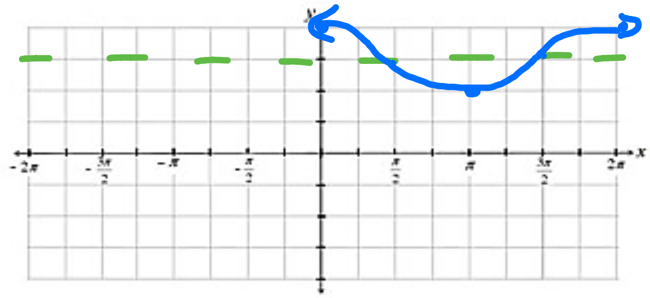
9. $f(\theta) = 0.5sin\theta$

Amplitude: $\frac{1}{2}$　　　　period: 2π



10. $f(\theta) = cos\theta + 3$

Amplitude: 1　　　　period: 2π



Use the given information to create a sine function.

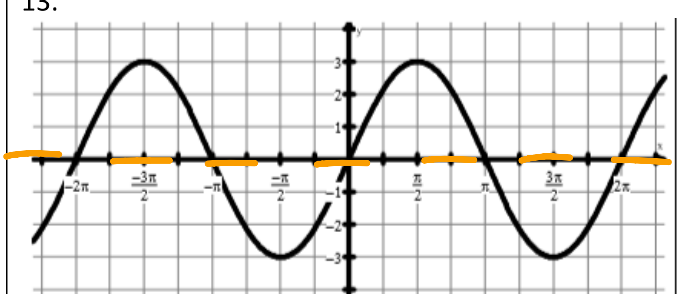
11.
Amplitude: 5
Period: 2π
Vertical Shift: down 4

$f(x) = 5\sin x - 4$

12.
Amplitude: $\frac{1}{5}$
Period: 2π
Vertical Shift: up 15

$f(x) = \frac{1}{5}\sin x + 15$

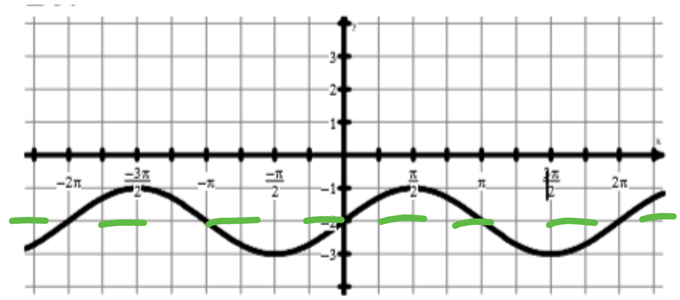
13.



VS: 0
a: 3

$\boxed{f(x) = 3\sin x}$

14.



VS: −2
a: 1

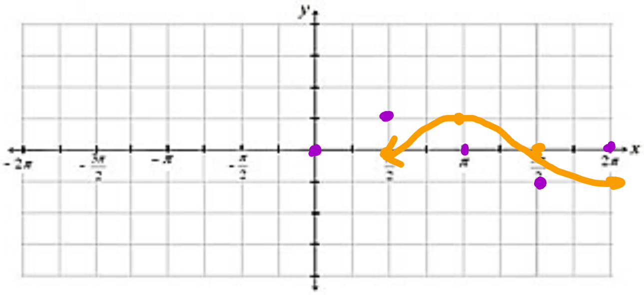
$\boxed{f(x) = \sin x - 2}$

Phase Shifts and Period Changes

Phase Shifts

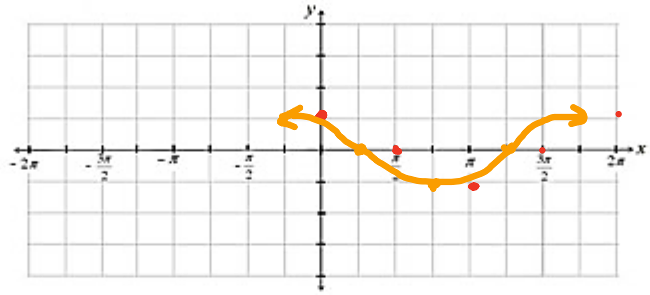
Ex. $f(\theta) = \sin\left(\theta - \frac{\pi}{2}\right)$

Amplitude: 1 period: 2π



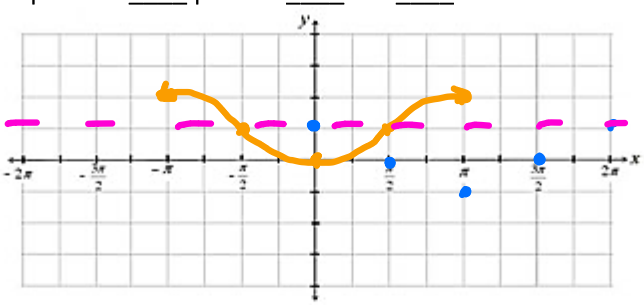
Ex. $f(\theta) = \cos\left(\theta + \frac{\pi}{4}\right)$ ←

Amplitude: 1 period: 2π



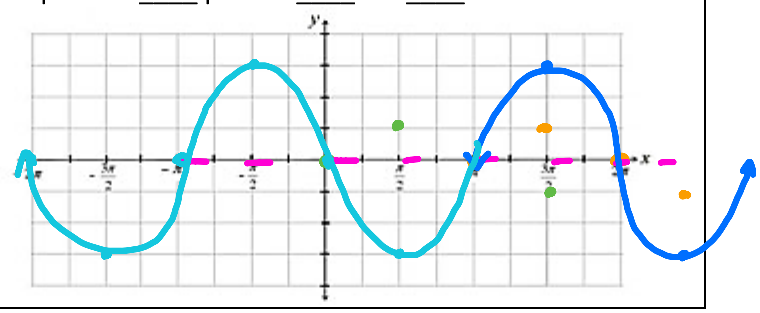
Ex. $f(\theta) = \cos(\theta + \pi) + 1$ ← π

Amplitude: 1 period: 2π VS: 1



Ex. $f(\theta) = 3\sin(\theta - \pi)$ → π

Amplitude: 3 period: 2π VS: 0

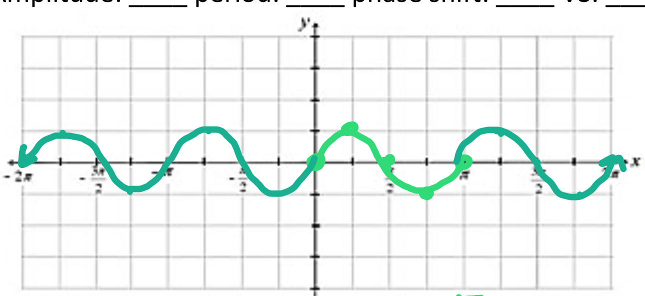


Period Changes

Period $= \frac{2\pi}{b} = \frac{2\pi}{2}$

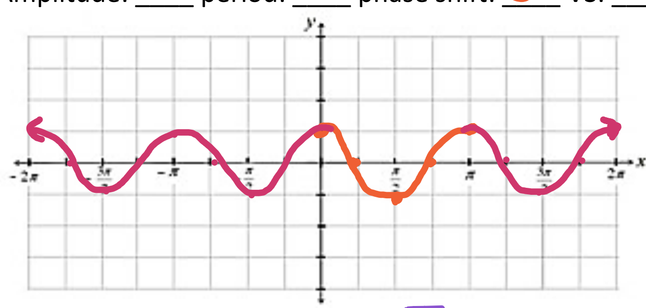
Ex. $f(\theta) = \sin 2\theta$

Amplitude: 1 period: π phase shift: 0 VS: 0



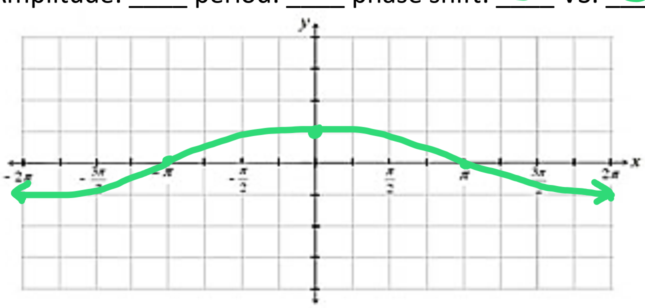
Ex. $f(\theta) = \cos 2\theta$

Amplitude: 1 period: π phase shift: 0 VS: 0



Ex. $f(\theta) = \cos \frac{1}{2}\theta$ $\frac{2\pi}{\frac{1}{2}}$

Amplitude: 1 period: 4π phase shift: 0 VS: 0



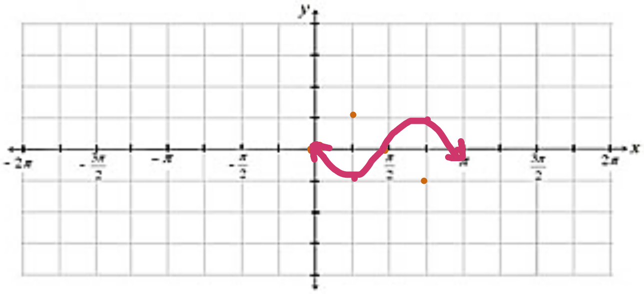
Ex. $f(\theta) = -\sin \frac{1}{2}\theta$ $\frac{2\pi}{\frac{1}{2}}$

Amplitude: 1 period: 4π phase shift: 0 VS: 0

Practice 2

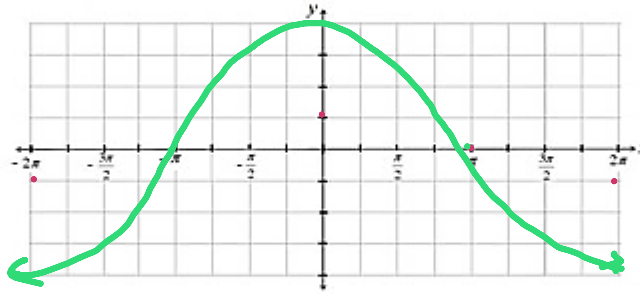
1. $f(\theta) = -sin2\theta$

Amplitude: 1 period: π phase shift: 0 VS: 0



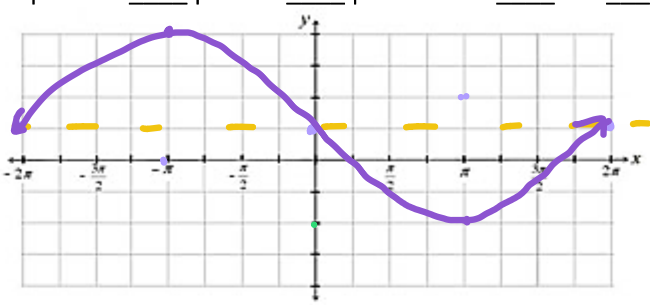
2. $f(\theta) = 4\cos\frac{1}{2}\theta$

Amplitude: 4 period: 4π phase shift: 0 VS: 0



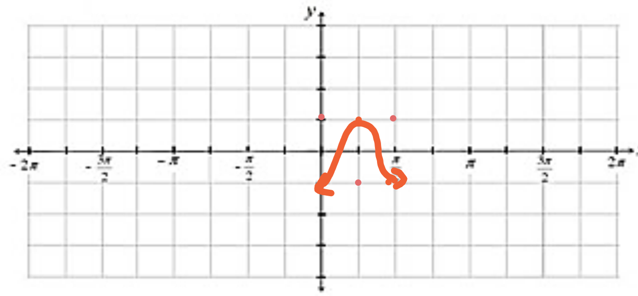
3. $f(\theta) = -3\sin\frac{1}{2}\theta + 1$

Amplitude: 3 period: 4π phase shift: 0 VS: 1



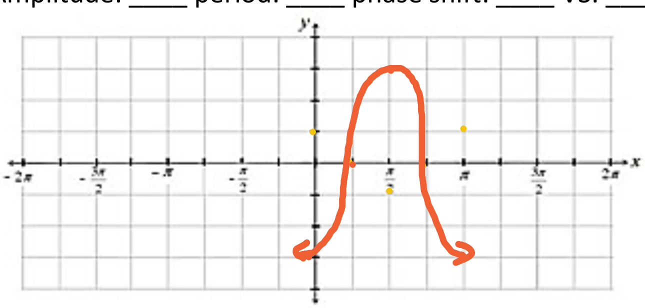
4. $f(\theta) = -cos4\theta$

Amplitude: 1 period: $\frac{\pi}{2}$ phase shift: 0 VS: 0



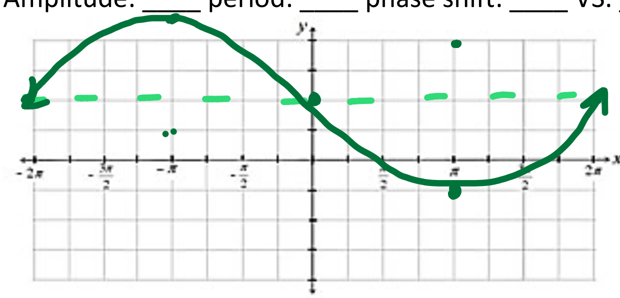
5. $f(\theta) = -3cos2\theta$

Amplitude: 3 period: π phase shift: 0 VS: 0



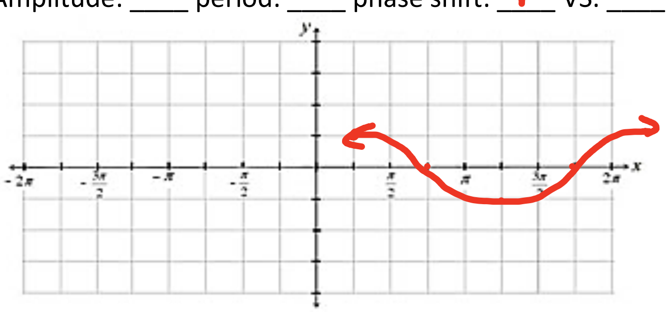
6. $f(\theta) = -3\sin\frac{1}{2}\theta + 2$

Amplitude: 3 period: 4π phase shift: 0 VS: 2



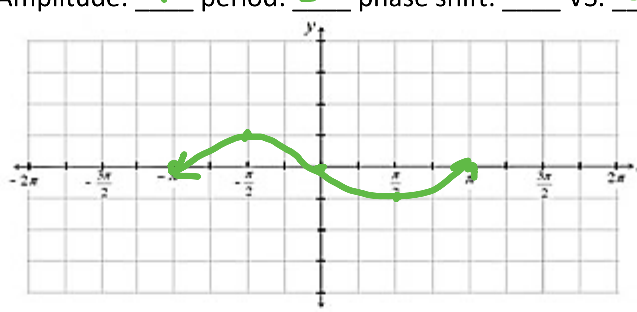
7. $f(\theta) = \cos(\theta - \frac{\pi}{4})$

Amplitude: 1 period: 2π phase shift: $\rightarrow \frac{\pi}{4}$ VS: 0



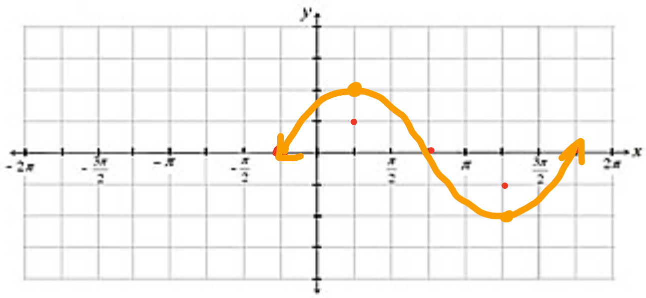
8. $f(\theta) = \sin(\theta + \pi)$

Amplitude: 1 period: 2π phase shift: $\leftarrow \pi$ VS: 0



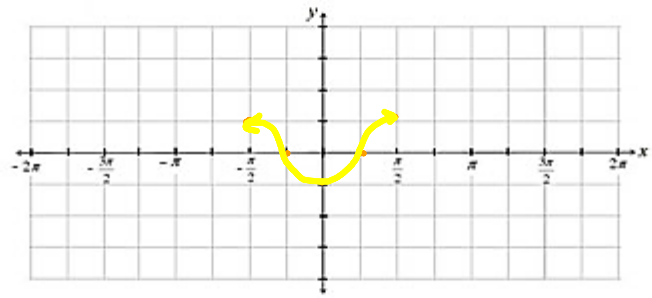
9. $f(\theta) = 2\sin(\theta + \frac{\pi}{4})$

Amplitude: 2 period: 2π phase shift: $\leftarrow \frac{\pi}{4}$ VS: 0



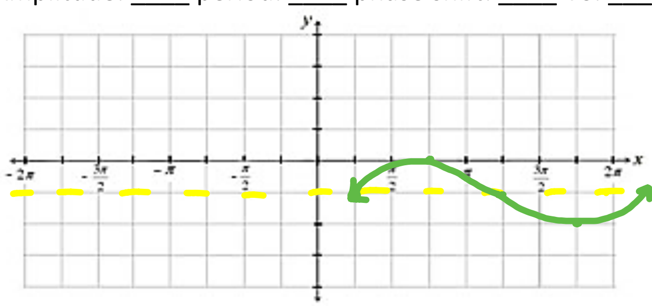
10. $f(\theta) = cos2(\theta + \frac{\pi}{2})$

Amplitude: 1 period: π phase shift: $\leftarrow \frac{\pi}{2}$ VS: 0



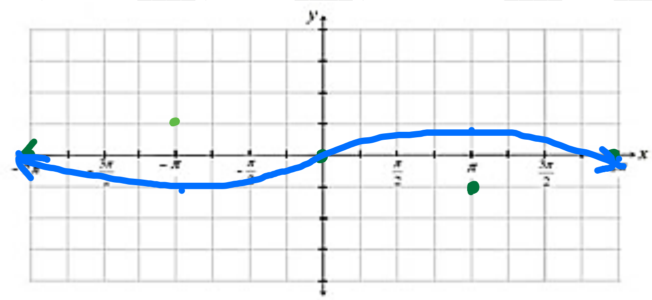
11. $f(\theta) = \sin\left(\theta - \frac{\pi}{4}\right) - 1$

Amplitude: 1 period: 2π phase shift: $\rightarrow \frac{\pi}{4}$ VS: -1



12. $f(\theta) = -\cos\frac{1}{2}(\theta + \pi)$

Amplitude: 1 period: 4π phase shift: $\leftarrow \pi$ VS: 0



Use the given information to create a sine function.

$f(\theta) = a \sin b(\theta - h) + K$

13.
Amplitude: $\frac{1}{5}$
Period: $\frac{\pi}{10}$ $b = 20$
Vertical Shift: up 15

$f(x) = \frac{1}{5}\sin 20x + 15$

14.
Amplitude: 5
Period: 4π
Vertical Shift: up 4

$f(x) = 5\sin\frac{1}{2}\theta + 4$

15.
Amplitude: 5
Period: $\frac{\pi}{6}$ $b = 12$
Phase Shift: right $\frac{\pi}{24}$
Vertical Shift: up 8

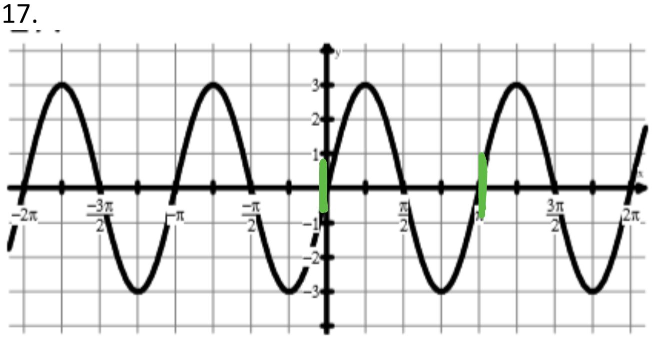
$f(x) = 5\sin 12(\theta - \frac{\pi}{24}) + 8$

16.
Amplitude: 2
Period: $\frac{3\pi}{2}$ $b = \frac{4}{3}$
Phase Shift: left $\frac{5\pi}{9}$
Vertical Shift: down 14

$\text{period} = \frac{2\pi}{b}$

$f(x) = 2\sin\frac{4}{3}(x + \frac{5\pi}{9}) - 14$

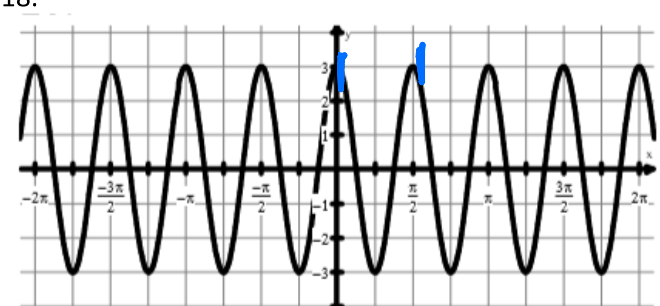
17.



a: 3 period: π $b = 2$

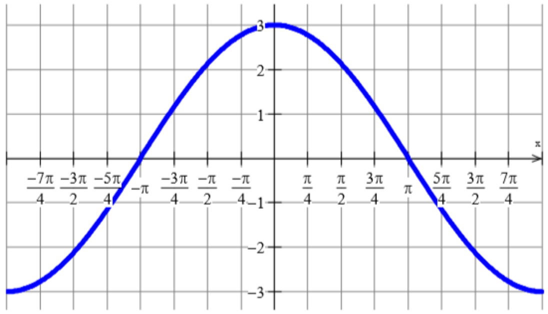
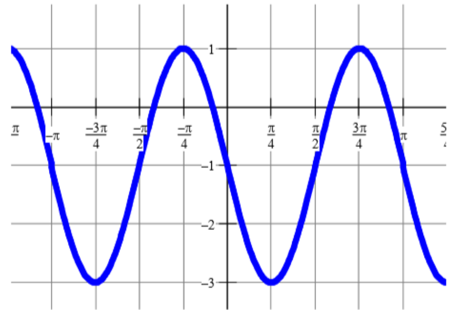
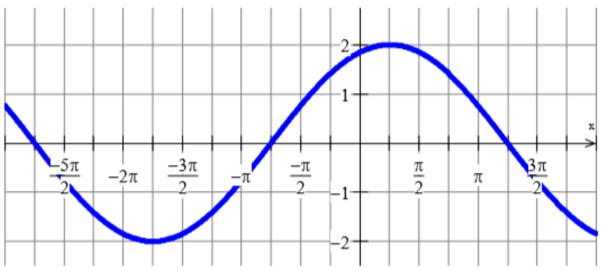
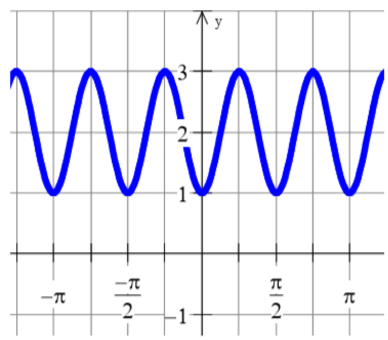
$\boxed{f(x) = 3\sin 2x}$

18.



a: 3 period: $\frac{\pi}{2}$ $b = 4$

$\boxed{f(x) = 3\cos 4x}$

19. 	20. 
21. 	22. 

23. The ferris wheel at the carnival is 150 feet tall. The bottom cart sits 4 feet off the ground. It takes 3 minutes to make 5 rotations.

1. What represents the amplitude?

2. What represents the midline?

3. How can you find b if you know the period?

4. Would this be a sine or cosine equation if you started at the bottom?

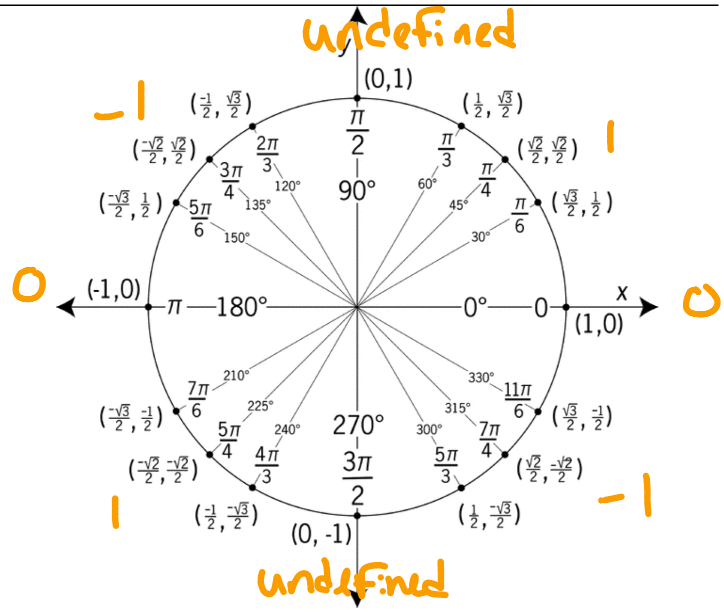
5. Write an equation for the ferris wheel ride if you start at the bottom and then end up at the bottom.

6. Make this into a sine equation. (you must start at a different spot on the ferris wheel which makes this equation have a phase shift.

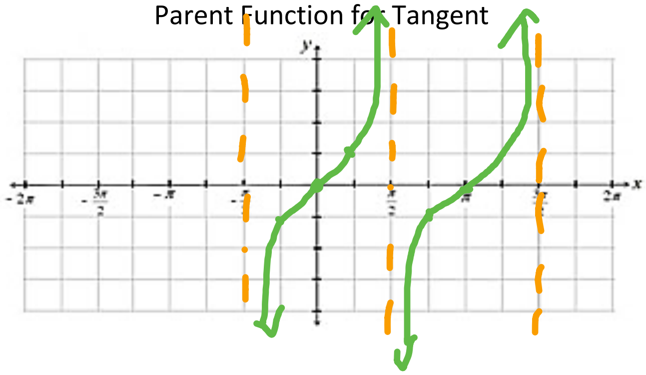
Tangent Notes

θ	$f(\theta)$
0	0
$\frac{\pi}{4}$	1
$\frac{\pi}{2}$	U
$\frac{3\pi}{4}$	-1
π	0

θ	$f(\theta)$
$\frac{5\pi}{4}$	1
$\frac{3\pi}{2}$	U
$\frac{7\pi}{4}$	-1
2π	0



Parent Function for Tangent



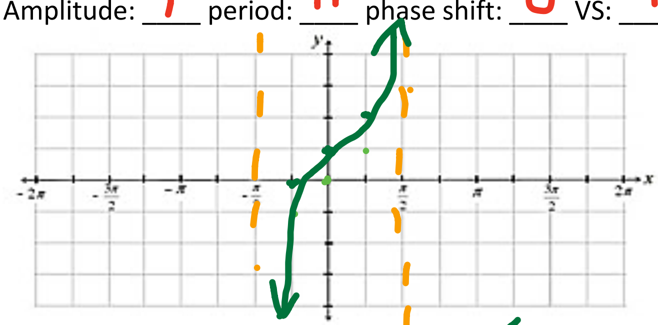
Standard equation:

$f(\theta) = a \cdot \tan\big(b(\theta - h)\big) + k$

$period = \frac{\pi}{b}$

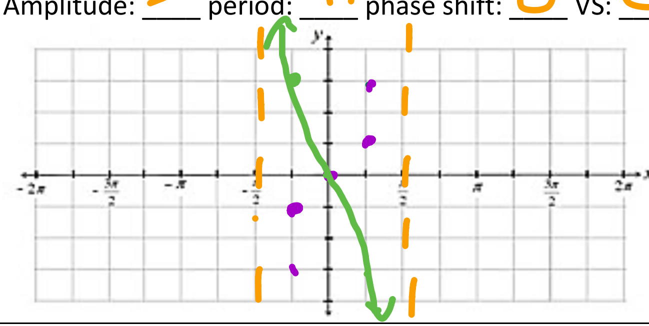
Ex. $f(\theta) = tan\theta + 1$

Amplitude: 1 period: π phase shift: 0 VS: 1



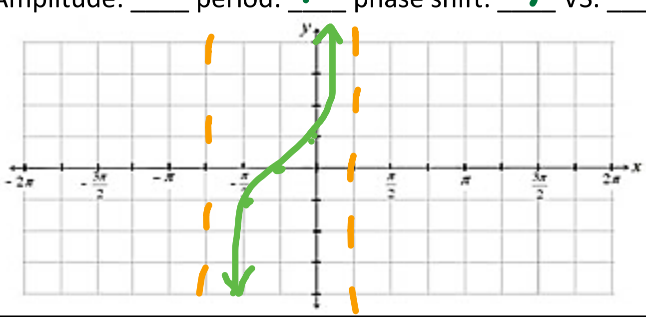
Ex. $f(\theta) = -3tan\theta$

Amplitude: 3 period: π phase shift: 0 VS: 0



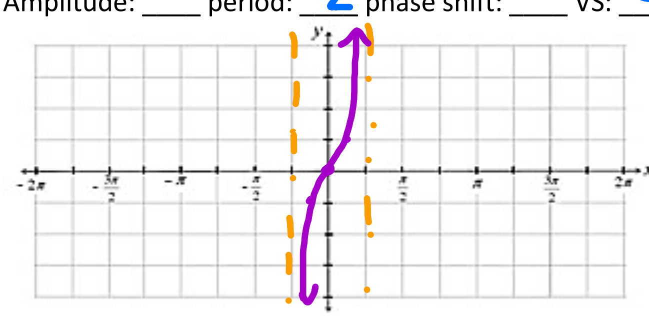
Ex. $f(\theta) = tan\,(\theta + \frac{\pi}{4})$

Amplitude: 1 period: π phase shift: $\frac{\pi}{4}$ VS: 0



Ex. $f(\theta) = tan2\theta$

Amplitude: 1 period: $\frac{\pi}{2}$ phase shift: 0 VS: 0

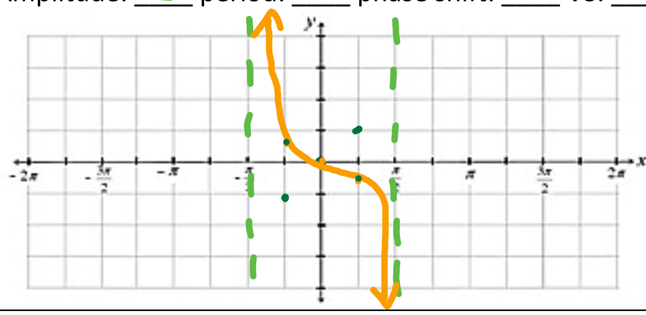


period $= \frac{\pi}{2}$

Practice

1. $f(\theta) = -\frac{1}{2} tan\theta$

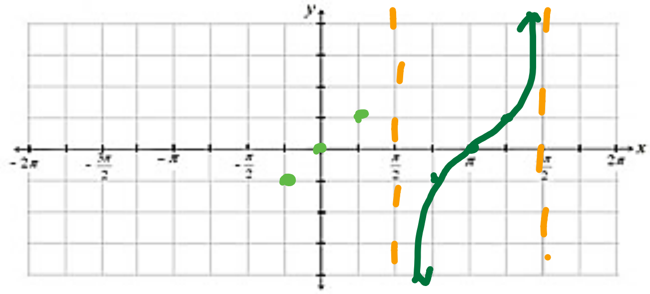
Amplitude: $\frac{1}{2}$ period: π phase shift: 0 VS: 0



2. $f(\theta) = \tan(\theta - \pi)$

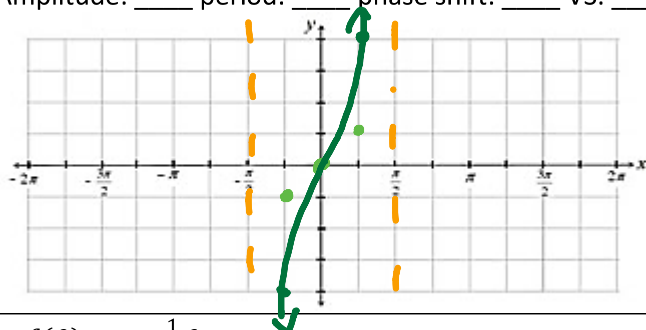
→π

Amplitude: 1 period: 0 phase shift: π VS: 0



3. $f(\theta) = 4tan\theta$

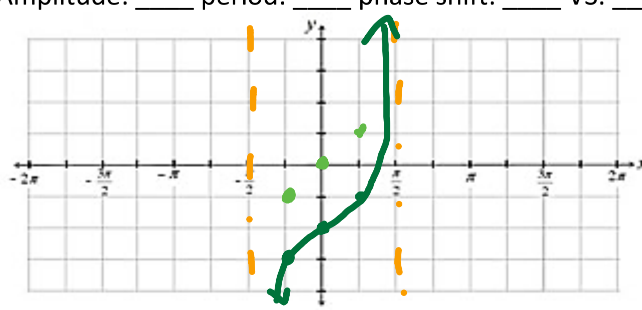
Amplitude: 4 period: π phase shift: 0 VS: 0



4. $f(\theta) = tan\theta - 2$

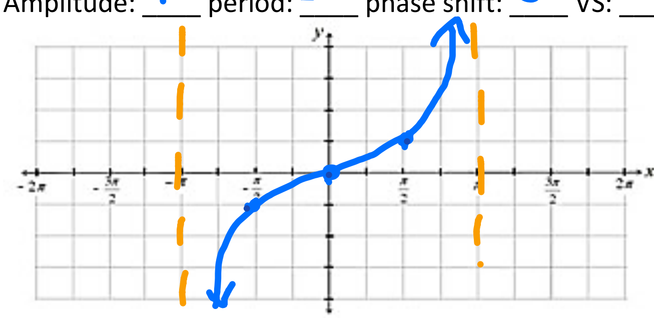
↓

Amplitude: 1 period: π phase shift: 0 VS: -2



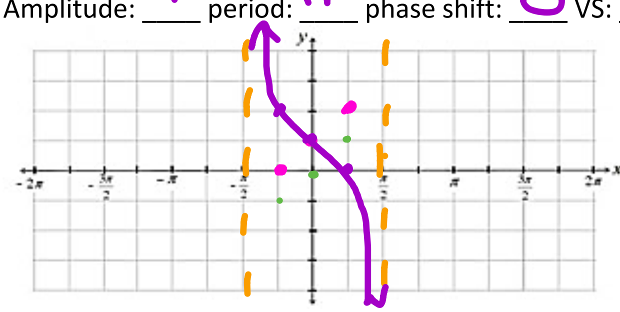
5. $f(\theta) = \tan\frac{1}{2}\theta$

Amplitude: 1 period: 2π phase shift: 0 VS: 0



6. $f(\theta) = -tan\theta + 1$

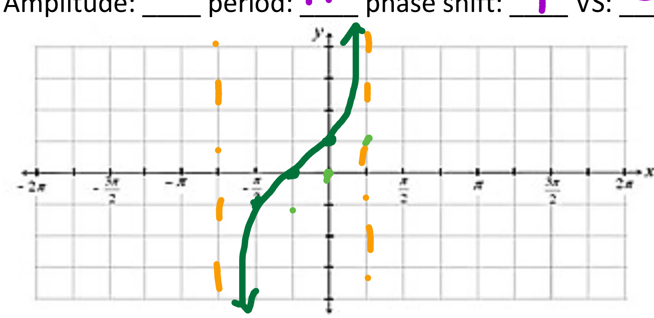
Amplitude: 1 period: π phase shift: 0 VS: 1



7. $f(\theta) = \tan(\theta + \frac{\pi}{4})$

←

Amplitude: 1 period: π phase shift: $\frac{\pi}{4}$ VS: 0



8. $f(\theta) = -2tan\theta$

Amplitude: 2 period: π phase shift: 0 VS: 0