Equations of Circles

Standard form: $(x-h)^2+(y-k)^2=r^2$

center: (h,k) radius $= r$

General form:

$$ax^2+by^2+cx+dy+e=0$$

Ex.1 Write the equation of a circle with center (3,-2) and a radius of 4.

h k

r

$(x-h)^2+(y-k)^2=r^2$

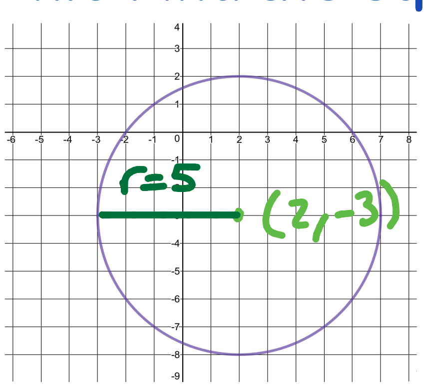
$$\boxed{(x-3)^2+(y+2)^2=16}$$

Ex.2 Find the coordinates of the center and measure of the radius.

$(x-6)^2+(y+3)^2=\sqrt{25}$

center: $(6,-3)$ $\boxed{r=5}$

Ex.3 Find the equation of the circle.



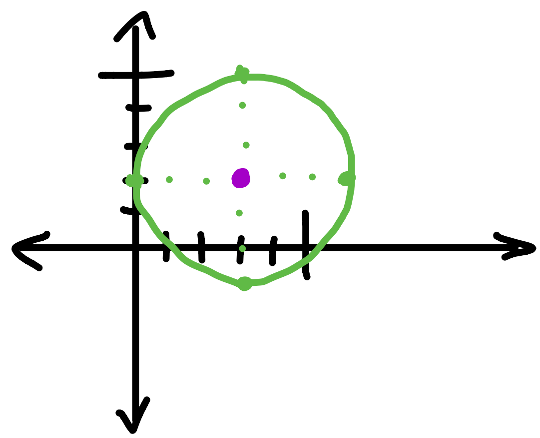
$(x-h)^2+(y-k)^2=r^2$

$$\boxed{(x-2)^2+(y+3)^2=25}$$

Ex.4 Graph the circle.

$$(x-3)^2+(y-2)^2=9$$

center: $(3,2)$ $r=3$



Converting from general to standard form.

1. (a) needs to be one.
2. Move the x and y terms together
3. Move (e) to the other side.
4. Complete the square.
5. factor the left side and simplify.

$ax^2+by^2+cx+dy+e=0$

Ex.5 $x^2+y^2-8x+7=0$

$x^2-8x+y^2+\cancel{7}=0$

$-7 \quad -7$

$x^2-8x\boxed{+(-4)^2}+y^2=-7\boxed{+(-4)^2}$

$a \cdot c$
$1 \cdot 16$
16
$1 \cdot 16$
$2 \cdot 8$
$4 \cdot 4$

$[x^2-8x+16]+y^2=9$

$(x-4)(x-4)+y^2=9$

$\boxed{(x-4)^2+y^2=9}$

Center: $(4,0)$

$r=3$

Ex.6 $x^2+y^2+4x-6y-3=0$

$x^2+4x+y^2-6y=3$

$(x+2)^2+(y-3)^2=3+2^2+3^2$

$\boxed{(x+2)^2+(y-3)^2=16}$

Ex.7 $\frac{2x^2}{2}+\frac{2y^2}{2}-\frac{16x}{2}+\frac{4y}{2}+\frac{20}{2}=\frac{0}{2}$

$x^2+y^2-8x+2y+10=0$

$x^2-8x+y^2+2y=-10$

$(x-4)^2+(y+1)^2=-10+4^2+1^2$

$$\boxed{(x-4)^2+(y+1)^2=7}$$

center: $(4,-1)$ radius $=\sqrt{7}$

Ex.8 standard to general form.

$(x-4)^2+(y+3)^2=36$

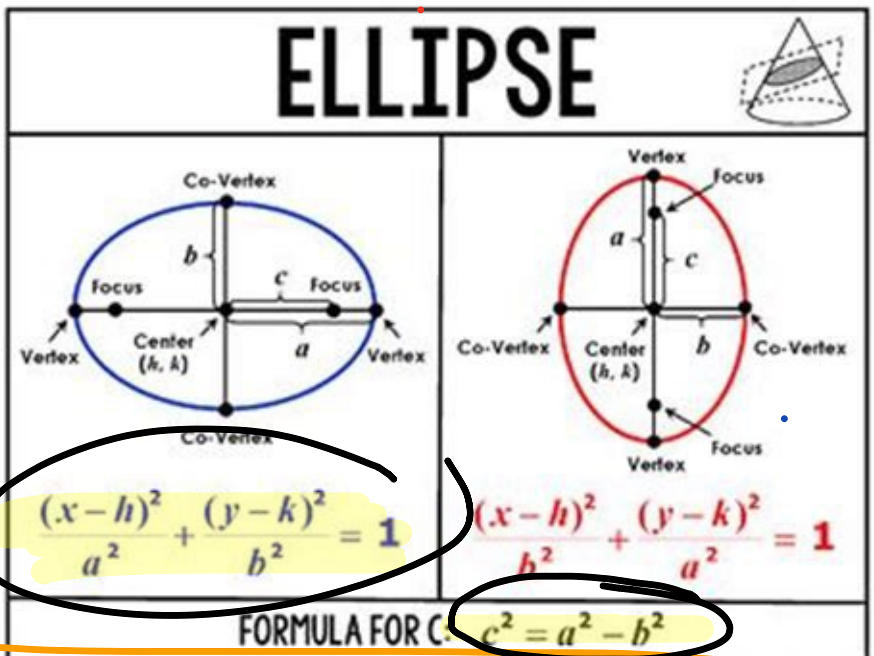
$(x-4)(x-4)+(y+3)(y+3)-36=0$

$x^2-4x-4x+16+y^2+3y+3y+9-36=0$

$$\boxed{x^2+y^2-8x+6y-11=0}$$

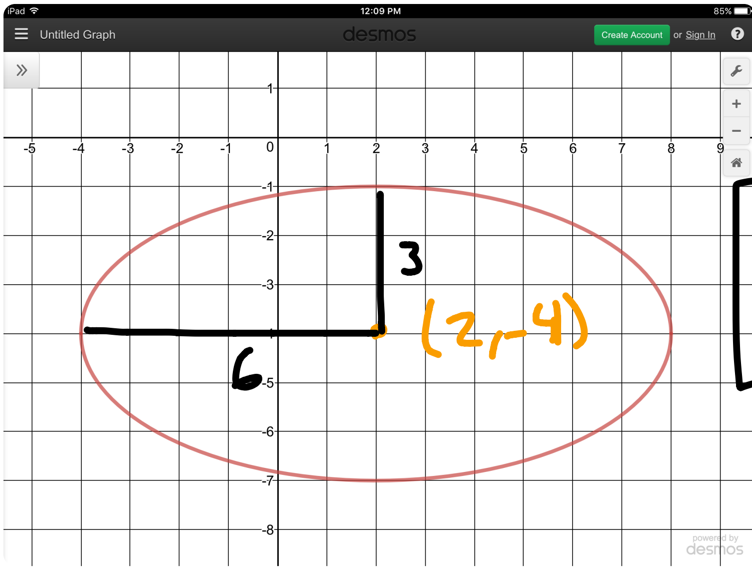
Ellipses

- Ellipses- is the set of all points in the plane, the sum of whose distances from two fixed points, called foci, is constant.
- Major axis- the longer line segment that contains the foci.
- Minor axis- the shorter segment.
- Vertices- the endpoints of each axis.



Center	(h,k)	(h,k)
Foci	$(h \pm c,k)$	$(h,k \pm c)$
Vertices	$(h \pm a,k)$	$(h,k \pm a)$
Co-vertices	$(h,k \pm b)$	$(h \pm b,k)$

Ex.1 write the equation of the ellipse.



$$\frac{(x-h)^2}{a^2}+\frac{(y-k)^2}{b^2}=1$$

$$\boxed{\frac{(x-2)^2}{36}+\frac{(y+4)^2}{9}=1}$$

Ex.2 Graph

$$\frac{(x+4)^2}{9}+\frac{(y-3)^2}{25}=1$$

Center: $(-4,3)$

$\boxed{(-4, 3\pm4)}$

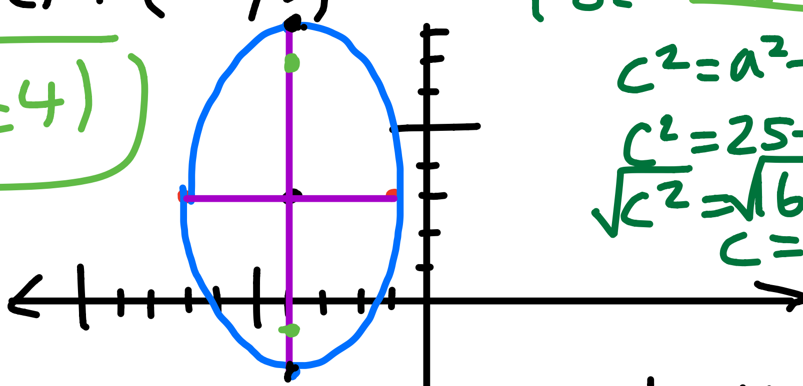
Foci: $\boxed{(-4,7) \quad (-4,-1)}$

$c^2=a^2-b^2$

$c^2=25-9$

$\sqrt{c^2}=\sqrt{16}$

$c=\pm4$



Vertices: $(-4,8)$
$(-4,-2)$

Co-vertices:
$(-1,3)$
$(-7,3)$

Major axis $=10$

Minor axis $=6$

Ex.3 Find the coordinates of the center, the foci, and the vertices.

$$25x^2+4y^2+100x-40y=-100$$

$$25x^2+100x+4y^2-40y=-100$$

$$25(x^2+4\cancel{x})+4(y^2-10\cancel{y})=-100$$

$$25(x+2)^2+4(y-5)^2=-100+25(2)^2+4(5)^2$$

$$\frac{25(x+2)^2}{100}+\frac{4(y-5)^2}{100}=\frac{10\cancel{0}}{\cancel{1}00}$$

$$\boxed{\frac{(x+2)^2}{4}+\frac{(y-5)^2}{25}=1}$$

Ex.4 $4x^2 - 40x + 9y^2 + 36y + 100 = 0$

$$4(x^2 - 10x) + 9(y^2 + 4y) = -100$$

$$4(x-5)^2 + 9(y+2)^2 = -100 + 4(5)^2 + 9(2)^2$$

$$\frac{4(x-5)^2}{36} + \frac{9(y+2)^2}{36} = \frac{\cancel{36}}{\cancel{36}}$$

$$\boxed{\frac{(x-5)^2}{9} + \frac{(y+2)^2}{4} = 1}$$

Hyperbolas

Is a set of all points in the plane in which the difference of the distances from the two distinct fixed points.

$$\frac{(x-h)^2}{a^2} - \frac{(y-k)^2}{b^2} = 1 \qquad \frac{(y-k)^2}{a^2} - \frac{(x-h)^2}{b^2} = 1$$

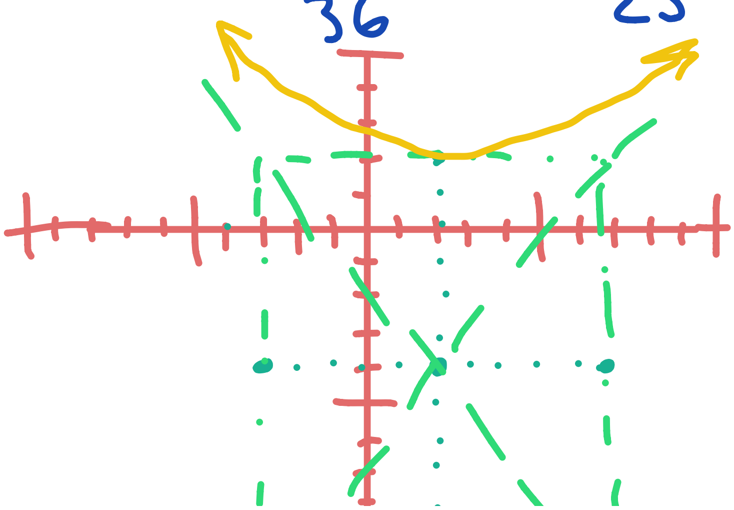
$$b^2 = c^2 - a^2$$

Equation of the asymptote

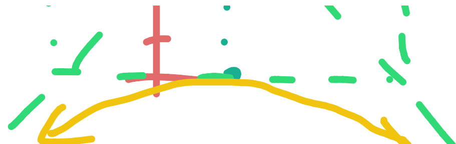
$$y-k = \pm \frac{b}{a}(x-h) \qquad y-k = \pm \frac{a}{b}(x-h)$$

Ex.1 Graph the Equation

$$\frac{(y+4)^2}{36} - \frac{(x-2)^2}{25} = 1$$



center: $(2, -4)$



Ex.2 write the equation in standard form then graph.

$$9x^2-4y^2-54x+40y-55=0$$

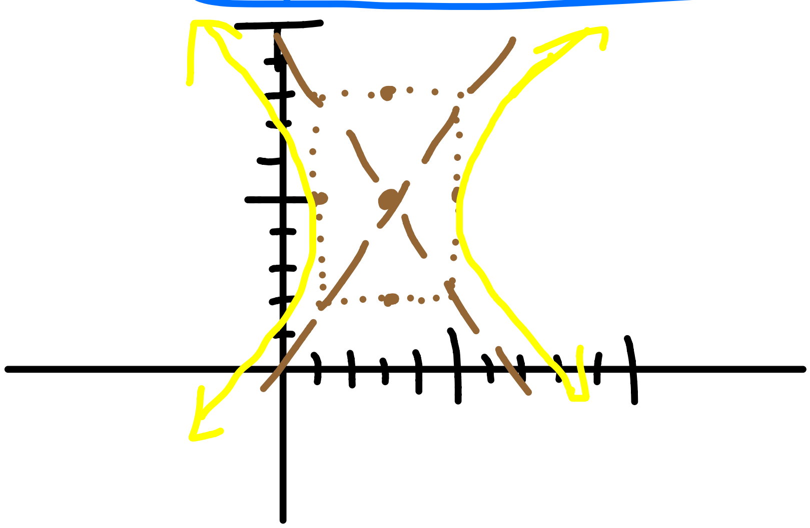
$$9x^2-54x-4y^2+40y=55$$

$$9(x^2-6x)-4(y^2-10y)=55$$

$$9(x-3)^2-4(y-5)^2=55+9(3)^2-4(5)^2$$

$$\frac{9(x-3)^2}{36}-\frac{4(y-5)^2}{36}=\frac{36}{36}$$

$$\boxed{\frac{(x-3)^2}{4}-\frac{(y-5)^2}{9}=1}$$



Parabolas

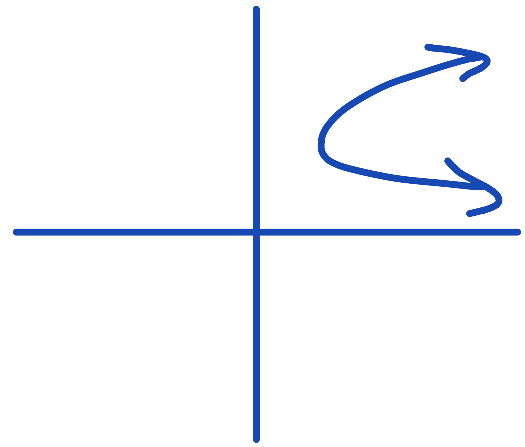
$(y-k)^2 = 4p(x-h)$

Vertex: (h,k)

focus: $(h+p,k)$

axis of symmetry: $y=k$

directrix $x=h-p$



- When p is positive, the parabola opens right
- When p is negative, the parabola opens left

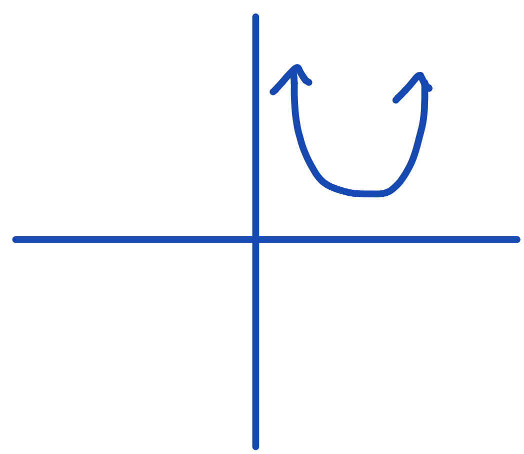
$(x-h)^2 = 4p(y-k)$

Vertex: (h,k)

focus: $(h,k+p)$

axis of symmetry: $x=h$

directrix: $y=k-p$



- When p is positive, the parabola opens up
- When p is negative, the parabola opens down

Ex.1 $y^2 = 8x + 48$

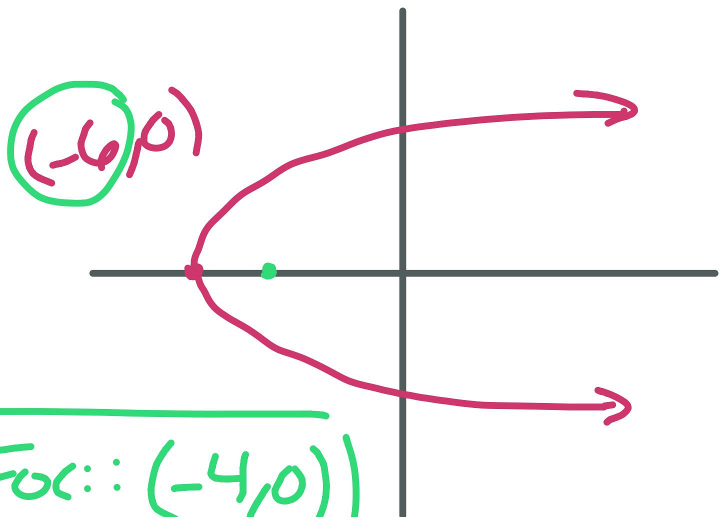
$y^2 = \boxed{8}(x+6)$

$(y-k) = \boxed{4p}(x-h)$

Center: $(-6,0)$

$\frac{8}{4} = \frac{\cancel{4}p}{\cancel{4}}$

$p = 2$



Foci: $(-4,0)$

Ex.2 $2x^2 - 8x + \cancel{y} \cancel{+6} = 0$ $\quad (x-h)^2 = 4p(y-k)$

$\cancel{-y} \cancel{-6} \quad -y-6$

$2x^2 - 8x = -y - 6$

$2(x^2 - 4x) = -y - 6$

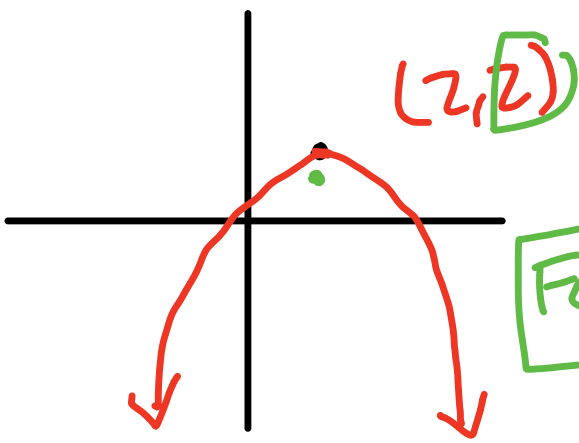
$2(x-2)^2 = -y - 6 + 2(2)^2$

$2(x-2)^2 = -y + 2$

$\frac{\cancel{2}(x-2)^2}{\cancel{2}} = \frac{-1(y-2)}{2}$

$(x-\underline{2})^2 = \boxed{-\frac{1}{2}}(y-\underline{2})$

Center: $(2,2)$



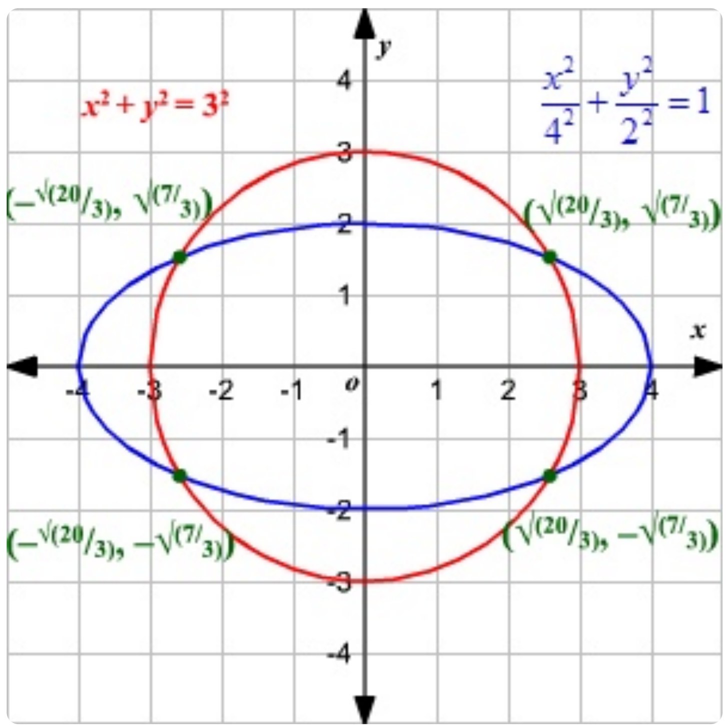
$\frac{4p}{4} = \frac{-\frac{1}{2}}{4}$

$p = -\frac{1}{8}$

$\boxed{\text{Foc: } (2, \tfrac{15}{8})}$

Systems of Equations

- A solution of a nonlinear system in two variables is an ordered pair of real numbers that satisfies both equations in the system.



Substitution

1. Solve one of the equations for one of the variables.
2. Substitute the expression into the other equation.
3. Solve the resulting equation.
4. Back substitute the obtained values into the original equations.
5. Check the proposed solutions in both equations.

Ex.1

$$\begin{cases} x^2 = 2y + 10 \\ 3x - y = 9 \end{cases}$$

$$-9 \; \cancel{+y} \quad \cancel{-9} \; +y$$

$$3x - 9 = y$$

$$x = 2$$

$$x = 4$$

$$\boxed{\begin{matrix} (2,-3) \\ (4,3) \end{matrix}}$$

$$x^2 = 2(3x - 9) + 10$$

$$x^2 = 6x - 18 + 10$$

$$x^2 = 6\cancel{x} - \cancel{8}$$

$$-6x + 8 \quad -6x \quad +8$$

$$x^2 - 6x + 8 = 0$$

$$(x-2)(x-4) = 0$$

$$x - 2 = 0 \qquad x - 4 =$$

Ex.2

$$\begin{cases} x - y = 3 \\ (x-2)^2 + (y+3)^2 = 4 \end{cases}$$

$$+y \qquad +y$$

$$x = y + 3$$

$$(y+3-2)^2 + (y+3)^2 = 4$$

$$(y+1)^2 + (y+3)^2 = 4$$

$$(y+1)(y+1) + (y+3)(y+3) = 4$$

$$y^2 + 2y + 1 + y^2 + 6y + 9 = 4$$

$$\frac{2y^2}{2} + \frac{8y}{2} + \frac{6}{2} = \frac{0}{2}$$

$$y = -1$$

$$y = -3$$

$$y^2 + 4y + 3 = 0$$

$$(y+1)(y+3)=0$$

$(0,-3)$

Elimination

1. Write both equations in the form
   $Ax^2+By^2=C$
2. If necessary, multiply either equation or both by appropriate numbers so that the sum of the x^2 coefficients or sum of the y^2 coefficients is zero.
3. Add the equations and solve for the remaining variable.
4. Back substitute and find the values of the other variable.
5. Check.

Ex.3
$$\begin{cases} 4x^2+y^2=13 \\ x^2+y^2=10 \end{cases}$$

$$4x^2+y^2=13$$
$$-x^2-y^2=-10$$
$$3x^2+0=3$$

$$\frac{3x^2}{3}=\frac{3}{3}$$
$$\sqrt{x^2}=\sqrt{1}$$
$$x=\pm 1$$

$$(1)^2+y^2=10$$
$$\sqrt{y^2}=\sqrt{9}$$
$$y=\pm 3$$

$(1,3)$
$(1,-3)$
$(-1,3)$
$(-1,-3)$

Ex.4

$$\begin{cases} y = x^2 + 3 \\ x^2 + y^2 = 9 \end{cases}$$

$$-x^2 \qquad -x^2$$

$$-x^2 + y = 3$$

$$y^2 + y = 12$$

$$-12 \quad -12$$

$$y^2 + y - 12 = 0$$

$$(y-3)(y+4) = 0$$

$$y = 3 \qquad y = \cancel{-4}$$

$$\boxed{(0,3)}$$

$$-4 = x^2 + \cancel{3}$$

$$-3 \qquad -\cancel{3}$$

$$\sqrt{-7} = \sqrt{x^2}$$

$$x = \pm i\sqrt{7}$$

$$\cancel{3} = x^2 + \cancel{3}$$

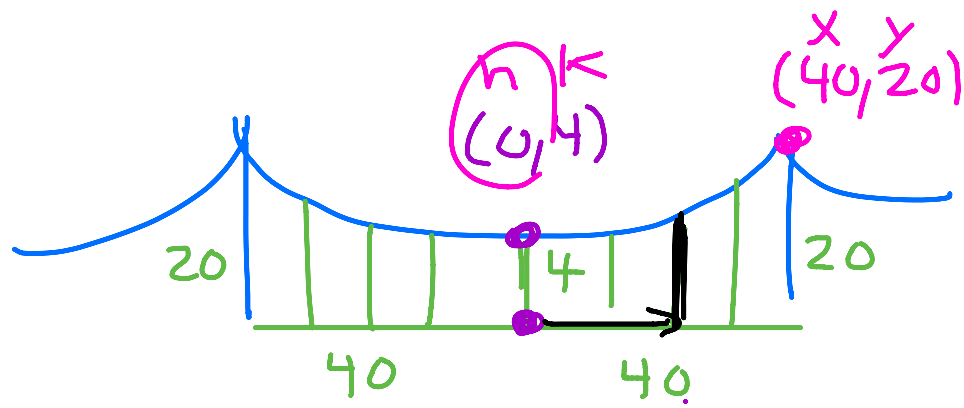
$$-\cancel{3} \qquad -\cancel{3}$$

$$\sqrt{x^2} = \sqrt{0}$$

$$x = 0$$

Ellipse word problems

Ex.1 The main cables of a suspension bridge are 20 meters above the road at the towers and 4 meters above the road at the center. The road is 80 meters long. Vertical cables are spaced every 10 meters. The main cables hang in the shape of a parabola. Find the equation of the parabola. Then, determine how high the main cable is 20 meters from the center.



$$\boxed{(x)^2=100(y-4)}$$

$x=20$

$$20^2=100(y-4)$$

$$\frac{400}{100}=\frac{\cancel{100}(y-4)}{\cancel{100}}$$

$$4=y-4$$

$$(x-h)^2=4p(y-k)$$

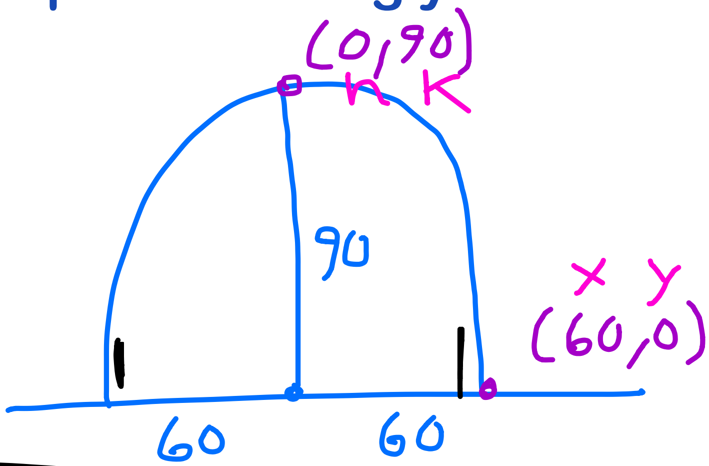
$$(40-0)^2=4p(20-4)$$

$$\frac{1600}{64}=\frac{\cancel{64}p}{\cancel{64}}$$

$p=25$

$\boxed{y-8)}$

Ex.2 The outer door of an airplane hanger is in the shape of a parabola. The door is 120 feet across and 90 feet high. Find an equation describing the door's shape. If you are 6 feet tall, how far must you stand from the edge of the door to keep from hitting your head?



$$\boxed{(x)^2 = -40(y-90)}$$

$(x-h)^2 = 4p(y-k)$

$(60-0)^2 = 4p(0-90)$

$\frac{3600}{-360} = \frac{-360p}{-360}$

$p = -10$

$y = 6$

$x^2 = -40(6-90)$

$\sqrt{x^2} = \sqrt{3360}$

$\boxed{x = \pm\ 57.97}$ ft